MATH 315: Fall 2024
Assignment 5
Due: Friday, September 27

I. Richardson Arms Race Model.

Complete the reading of Chapter 2 of our text. Note the **SUGGESTED PROJECTS** at the very end of the chapter and the **REFERENCES.**

Complete Problems 17, 19, 20, and 30 of Chapter 2.

II. The Homicide Problem and the "Trial of the Twentieth Century"

Newton's Law of Cooling asserts that the rate at which an object cools is proportional to the difference between the temperature of the object and the temperature of the environment in which the object is immersed.

If V denotes the temperature of the object (victim) at time t, then Newton's law takes the form

$$dV/dt = k\,(V - R)$$

where R is the temperature of the room and k is a proportionality constant dependent on the thermal properties of the object.

1) Verify that $V(t) = R + C\,e^{kt}$ is a solution to the differential equation.
2) By separating the variables and integrating, derive the solution to the differential equation given in Exercise 1.
3) Find the value of the constant C if the Victim's temperature is Vo at time $to = 0$.
4) If the temperature of the victim is V1 at some positive time t_1 then show that k has the value:

$$\frac{1}{t_1}\ ln\ \frac{V_1 - R}{V_o - R}$$

5) From the solution $V(t) = R + C\,e^{kt}$, show that if the victim has a temperature of V_N at time t_N then t_N can be found as

$$t_N = \frac{1}{k}\ ln\ \frac{V_N - R}{C}$$

6) Use the data in the Nicole Brown investigation with outside temperature 68°F to estimate the time of death. Assume that when Nicole died, she had a normal body temperature of 98.6°F. For simplicity, let t = 0 correspond to the time (12:30 am) at which the initial temperature was taken by Quincy.

7) (*Extra Credit*). Suppose that we do not know the room temperature R, but that we have the Victim's temperature (V_1, V_2, V_3) at three different times (t_1, t_2, t_3). How can we use this information to determine R, C and k? [Consider first the case that the three times are evenly spaced; that is, $t_3 - t_2 = t_2 - t_1$.