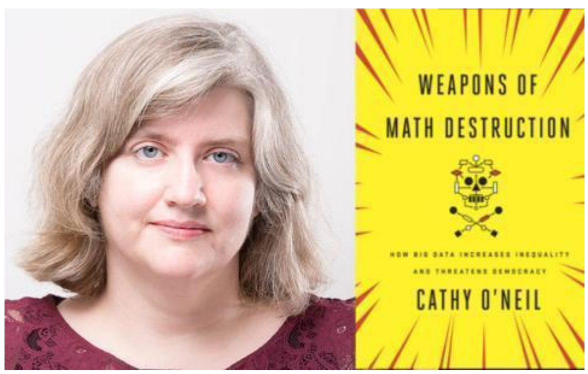
## Introduction To Probability Models

"Models are by their very nature, simplifications. No model can include all of the world's complexity or the nuance of human communication. Inevitably, some important information gets left out…

To create a model, then, we make choices about what's important enough to include, simplifying the world into a toy version that can be easily understood and from which we can infer important facts and actions. We expect it to handle only one job and accept that it will occasionally act like a clueless machine, one with enormous blind spots.

[But] sometimes these blind spots don't matter…

Cathy O'Neill
*Weapons of Math Destruction*
New York: Crown Press, 2016




**Introduction to Probability Models**

**Basic Calculations**

**Conditional Probability**

**Expected Value**

## Toss Fair Coin 5 Times

Pr(All 5 results are heads) = $\frac{1}{2^5} = \frac{1}{32}$

Pr(Exactly 4 heads) = $\binom{5}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = \frac{5}{32}$

Pr(Exactly 3 heads) = $\binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{10}{32}$

Pr(Exactly 2 Heads) = $\frac{10}{32}$

Note: This also equals Pr(Exactly 3 Tails) and Pr(Exactly 3 Heads)

Pr(**At Least** 3 Heads) = $\frac{10+5+1}{32} = \frac{1}{2}$

Pr(Exactly 3 Heads) + Pr(Exactly 4 Heads) + Pr(Exactly 5 Heads)

A = At least 3 Heads $\Rightarrow$ Pr(A) = $\frac{16}{32}$

B = At least 2 Heads $\Rightarrow$ Pr(B) = $\frac{26}{32}$

## Toss UNFAIR Coin 5 Times

**Probability of Head on Single Toss = 3/5**

$$\Pr(\text{All 5 results are Heads}) = \left(\frac{3}{5}\right)^5$$

$$\Pr(\text{Exactly 4 Heads}) = \binom{5}{4}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^1$$

$$\Pr(\text{Exactly 3 Heads}) = \binom{5}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2$$

$$\Pr(\text{Exactly } k \text{ Heads in } n \text{ Tosses}) = \binom{n}{k}\left(\frac{3}{5}\right)^k\left(\frac{2}{5}\right)^{n-k}$$

**Probability of Head on Single Toss = *p***

$$\Pr(\text{Exactly } k \text{ Heads in } n \text{ Tosses}) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

**Bernoulli Trials**

## Toss Fair Coin 5 Times

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