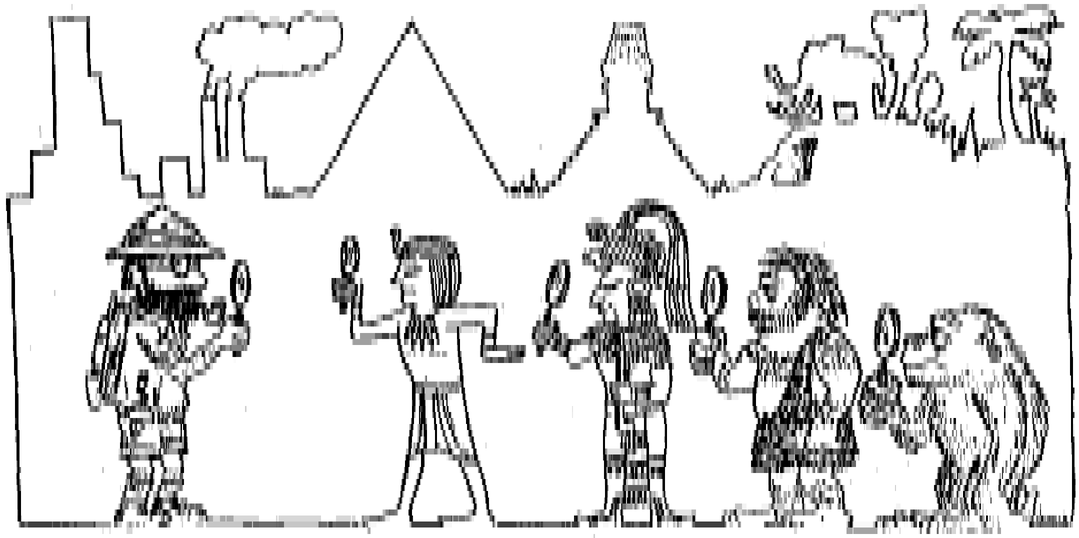
Mathematical Models in



Cultural Anthropology

Two Models of Cultural Stability in Anthropology

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Anthropology professor at SUNY Binghamton; pioneer in arguing for the use of mathematical models in anthropology.

Real World Situation: A system operating among Galla tribes in Ethiopia.
Tribal life is structured through five "age grades."
Each male moves through age grade system, spending 8 years successively in each grade.
A son enters grade one when his father retires from grade five – exactly 40 years after his father entered.

Example:
Your father enters at age 15.
His first son (you) is born when he's 25.
Second son is born when he's 30.
You have a son when you're 35.
Your brother has a son when he's age 20.

Time	*Event*
x	Your father enters at age 15
$x + 10$	**You are born.**
$x + 15$	**Your brother is born.**
$x + 35$	**Your nephew is born.**
$x + 40$	**You (30)** and **your brother (25)** enter; Father retires at age 55.
$x + 45$	**Your son is born.**
$x + 80$	**You retire at age 70.** **Your brother retires at age 65.** **Your son enters at age 35.** **Your nephew enters at age 45**

Ages at entrance: Generation 1: 15
Generation 2: 30, 25
Generation 3: 35, 45

Description of Roles

The Fundamental Problem:

It is evident that the stability of Galla communities is threatened by the arbitrary interval of 40 years that is interposed between generations. Since this interval is often greater than the actual chronological difference between generations, the ages of some of the people in the grades may become progressively greater. This can result in humiliation and incongruity. An old man, entering the first grade, would be required to abstain from sexual activity and to wander around with its youthful members begging food. Further, if he should die before attaining the higher grades, important governmental offices may go unfilled.

Definition: A system is ***stable*** if it tends to maintain a realistic relationship between age and role behavior.

Axiom: A realistic relationship can be maintained if between any arbitrary number of generations, the ages at which an ancestor and his distant offspring enter the first grade are the same.

Under which conditions will this criterion be met?

A Deterministic Model

For generation i, let

A_i = the age at which a man enters first grade.

P_i =age which he becomes a father

Then at age of retirement, we have

$$\mathbf{A_i + 40 = P_i + A_{i+1}}$$

Start with $A_1 + 40 = P_1 + A_2$

So $A_2 = A_1 + 40 - P_1$ and hence $A_2 = A_1$ if and only if $P_1 = 40$.

<u>Two generation gap:</u>

$$A_3 = A_2 + 40 - P_2 = (A_1 + 40 - P_1) + (40 - P_2)$$

$$= A_1 + 2 * 40 - (P_1 + P_2)$$

Thus $A_3 = A_1$ exactly when $(P_1 + P_2)/2 = 40$.

A simple induction argument shows

$$A_{n+1} = A_1 + n(40) - (P_1 + P_2 + \ldots + P_n)$$

So $A_{n+1} = A_1$ if and only if

$(P_1 + P_2 + \ldots + P_n)/n = 40$

Remarks:

(1) $A_{n+2} = A_{n+1} + 40 - P_{n+1}$

$= (A_1 + 40n - (P_1 + \ldots + P_n)) + 40 - P_{n+1}$

$= A_1 + (n+1)40 - (P_1 + \ldots + P_n + P_{n+1})$

(2) If $(P_1 + P_2 + \ldots + P_n)/n > 40$,

then $A_{n+1} < A_1$

Conclusion: The system will be stable if average age of parenthood is 40.

Note: It is easy to check via census data if this condition holds.

Limitations of the Deterministic Model

(1) Deals only with 1-dimensional father-son links: ignores branching of descent lines representing siblings.

(2) The model transforms every given family into some point in the future when it may no longer exist.

Probabilistic Model

For stability, it is important that lower grades consist **largely** of adolescents.

What is critical are **relative** numbers, not the absolute count of members of different ages in a particular grade.

Hoffmann studies group of 240 males via a Markov Chain

Steps: Successive generations

States: Age at time of entrance:

S_1 : 13 – 19

S_2: 20 – 29

S_3: ≥ 30

		Son's State at Time of Initiation			
		S_1	S_2	S_3	***Total***
Father's State At initiation	S_1	10	25	30	**65**
	S_2	55	60	35	**150**
	S_3	5	15	5	**25**

Form Transition Matrix:

$$P = \begin{pmatrix} 10/65 & 25/65 & 30/65 \\ 55/150 & 60/150 & 35/150 \\ 5/25 & 15/25 & 5/25 \end{pmatrix} = \begin{pmatrix} .154 & .384 & .462 \\ .367 & .4 & .233 \\ .2 & .6 & .2 \end{pmatrix}$$

$$\vec{p}^{(0)} = (.25,.55,.20)$$

100 men about to enter: (25, 55, 20)

Then $\vec{p}^{(1)} = (.28, .44, .28)$

$$\vec{\mathbf{w}} = \lim_{n\to\infty} \vec{p}^{(n)} = \left(\frac{663}{2518}, \frac{1140}{2518}, \frac{715}{2518}\right) = (.263, .453, .284)$$

$$P = \begin{bmatrix} \frac{2}{13} & \frac{5}{13} & \frac{6}{13} \\ \frac{11}{30} & \frac{2}{5} & \frac{7}{30} \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \end{bmatrix}$$

$\mathbf{w} = (x, y, z)$

$\mathbf{w}P = \mathbf{w}$ becomes

$$(2/13)\,x + (11/30)\,y + (1/5)\,z = x$$
$$(5/13)\,x + (2/5)\,y + (3/5)\,z = y$$
$$(6/13)\,x + (7/30)\,y + (1/5)\,z = z$$

which we can rewrite as a set of homogeneous equations

$$-\frac{11}{13}x + \frac{11}{30}y + \frac{1}{5}z = 0$$
$$\frac{5}{13}x - \frac{3}{5}y + \frac{3}{5}z = 0$$
$$\frac{6}{13}x + \frac{7}{30}y - \frac{4}{5}z = 0$$

The Coefficient Matrix is

$$\begin{bmatrix} -\frac{11}{13} & \frac{11}{30} & \frac{1}{5} \\ \frac{5}{13} & -\frac{3}{5} & \frac{3}{5} \\ \frac{6}{13} & \frac{7}{30} & -\frac{4}{5} \end{bmatrix}$$

which has reduced row echelon form

$$\begin{bmatrix} 1 & 0 & -\frac{51}{55} \\ 0 & 1 & -\frac{228}{143} \\ 0 & 0 & 0 \end{bmatrix}$$

so

$$x = \frac{51}{55} z$$

$$y = \frac{228}{143} z$$

z has any value

But $x + y + z = 1$

$$\frac{51}{55} z + \frac{228}{143} z + z = 1$$

$$\frac{2518}{715} z = 1$$

$$x = \frac{715}{2518} = 0.263$$

$$y = \frac{1140}{2518} = 0.453$$

$$z = \frac{715}{2518} = 0.284$$

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