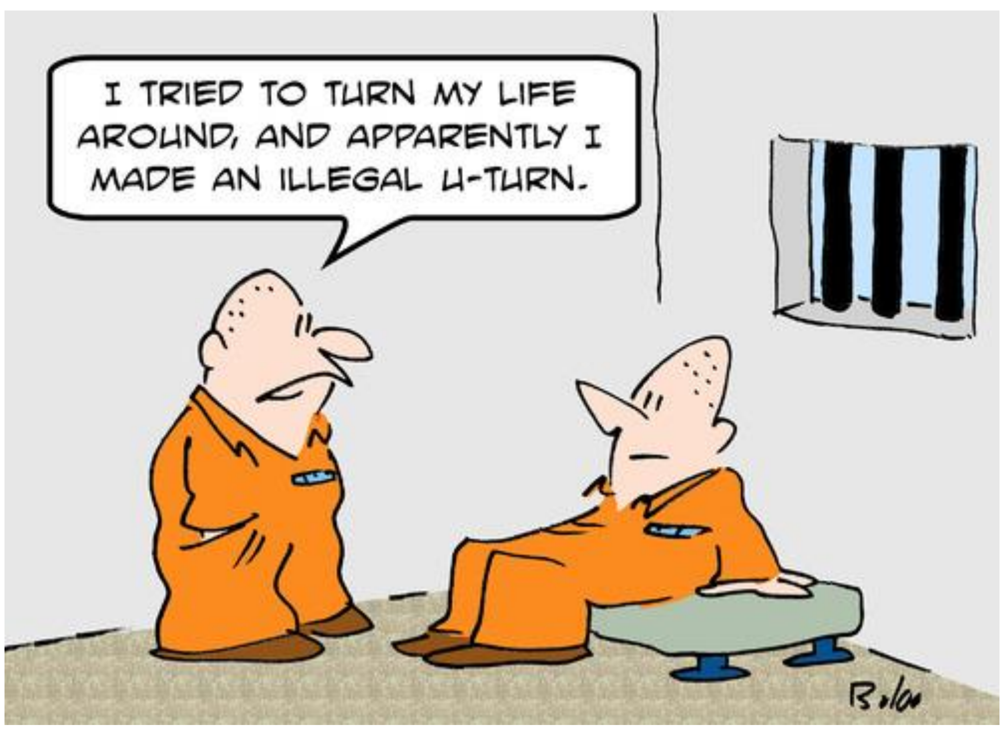
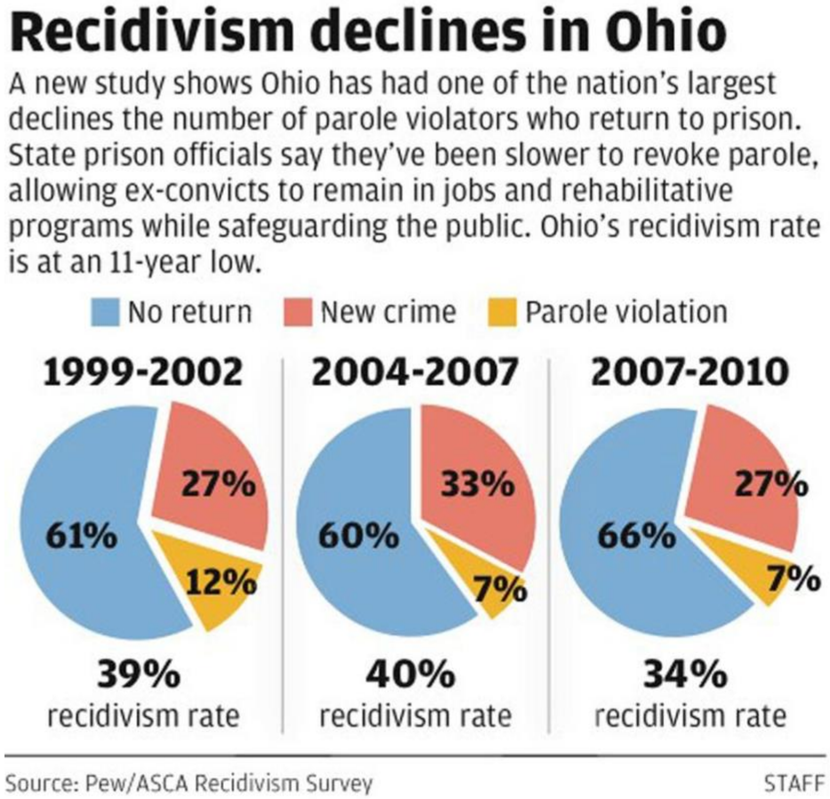
RECIDIVISM:

Part I





UNITED STATES SENTENCING COMMISSION

MODELING AND MEASURING RECIDIVISM

Notes by Olinick (Chapter 17)

Alfred Blumstein and Richard Larson, "Problems in Modeling and Measuring Recidivism," *Journal of Research in Crime and Delinquency*, Volume 8 (1971), 124 - 132.

David F. Greenberg, *Mathematical Criminology*, Rutgers University Press, 1979.

Michael Maltz, *Recidivism*, Academic Press, 1984.

United States Sentencing Commission, *Recidivism Among Federal Offenders: A Comprehensive Overview*, March 2016.

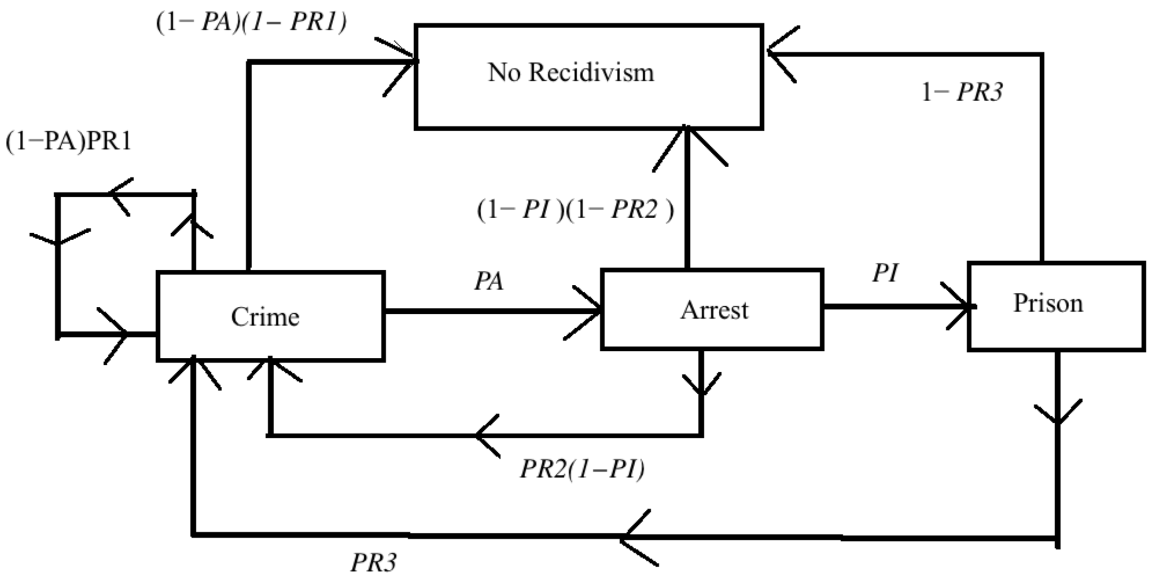
Definition: **Recidivism** is the tendency to relapse into a previous condition; especially a relapse into criminal behavior.

Recidivism can be measured by:

Arrests
Convictions
Imprisonments

A Markov Chain Model

States: **Crime**, **Arrest**, **Prison**, and **No Recidivism**



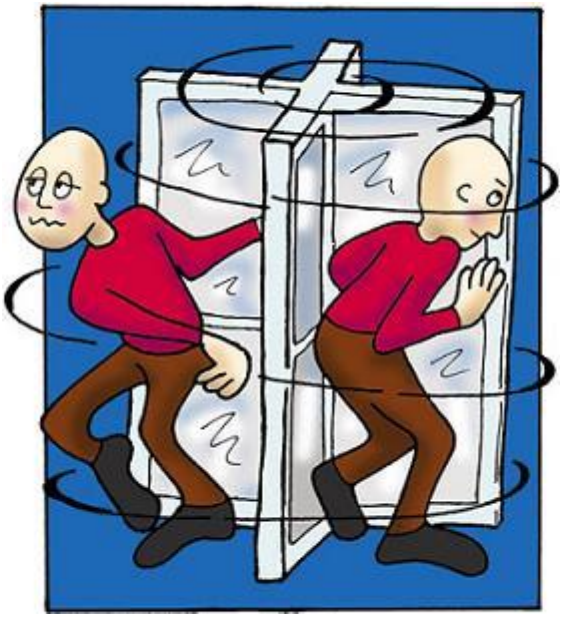
P_A = Probability that an offender is arrested after committing a crime

P_I = Probability that arrested offender is incarcerated

P_{R1}= Probability that unapprehended offender commits at least one more crime.

P_{R2}= Probability that offender who has been arrested but not incarcerated will commit at least one more crime.

P_{R3}= Probability that incarcerated offender commits at least one more crime after release.



Recidivism can be measured in at least 3 different ways:

Pr(C | C) = Probability (Offender will commit at least one more crime **given** he has just committed one)

Pr(A | A) = Probability (Offender will be arrested at least one more time **given** he has just been arrested).

Pr(I | I)= Probability (Offender will be incarcerated at least one more time **given** he has just been incarcerated)

"It's greatly reduced the rate of recidivism."

We shall show that

$$\Pr(C|C) = P_{R1}(1 - P_A) + P_A P_{R2}(1 - P_I) + P_A P_I P_{R3}$$

$$\Pr(A \mid A) = \frac{P_A [P_{R2} (1 - P_I) + P_I P_{R3}]}{1 - P_{R1}(1 - P_A)}$$

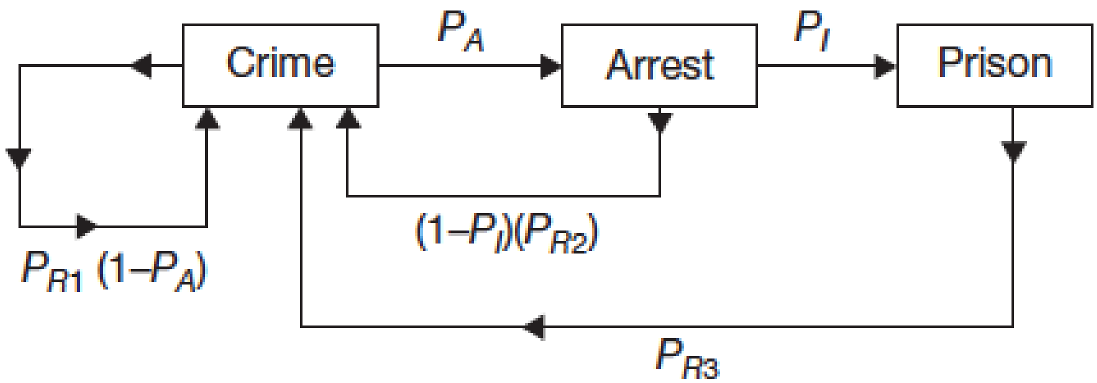
$$\Pr(I \mid I) = \frac{P_{R3} P_A P_I}{1 - P_{R1}(1 - P_A) - P_A (1 - P_I) P_{R2}}$$

Other conditional probabilities may be of interest:

$\Pr(C \mid A)$

$\Pr(A \mid I)$

Pr(C | C) = Prob (At least one more crime | He has just committed a crime)
= Prob (Return to **Crime** | Start at **Crime**)



There are 3 paths:

Crime ---$P_{R1}(1 - P_A)$---> **Crime**

Crime ---P_A---> **Arrest** ---$P_{R2}(1 - P_I)$---> **Crime**

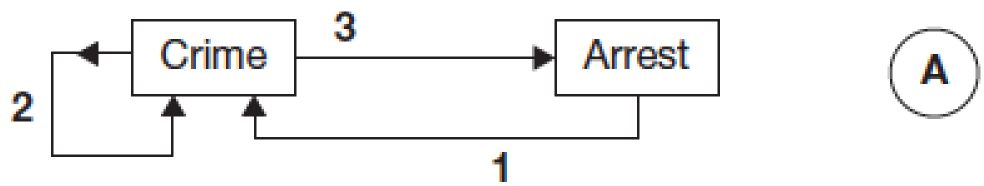
Crime ---P_A---> **Arrest** ---P_I---> **Prison** ---P_{R3}---> **Crime**

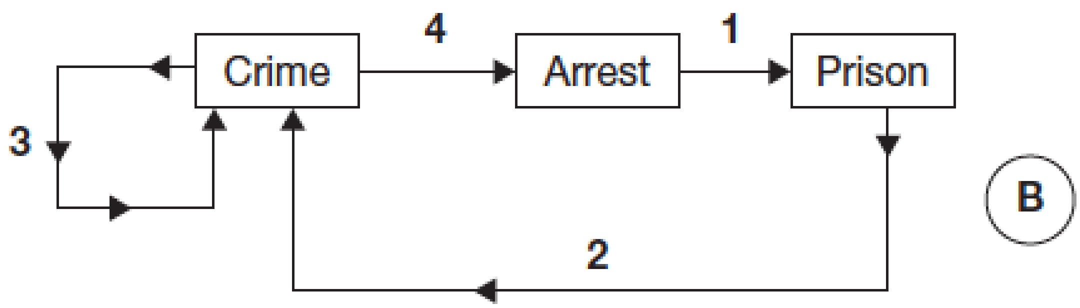
Pr(C | C) =

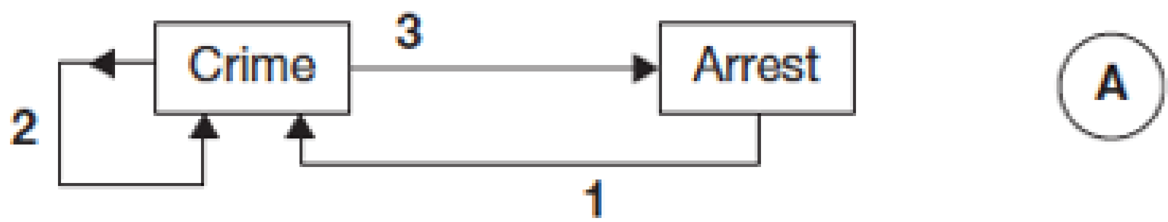
$$\mathbf{P_{R1}\,(1 - P_A) + P_A\,P_{R2}\,(1 - P_I) + P_A P_I P_{R3}}$$

Pr(A | A)

Prob (Return to **Arrest** | Start at **Arrest)**







A

Arrest □ Crime□ Arrest

Arrest □ Crime□ Crime□ Arrest

Arrest □□□ Crime □□□ Crime □□□ Crime □□□ Arrest
...

$$P_{R2}(1 - P_I)\ P_A$$
$$+ P_{R2}(1 - P_I)\ P_{R1}(1 - P_A)\ P_A$$
$$+ P_{R2}(1 - P_I)\ [\ P_{R1}(1 - P_A)\ ]^2\ P_A \ldots$$
$$+ P_{R2}(1 - P_I)\ [\ P_{R1}(1 - P_A)\ ]^k\ P_A + \ldots$$

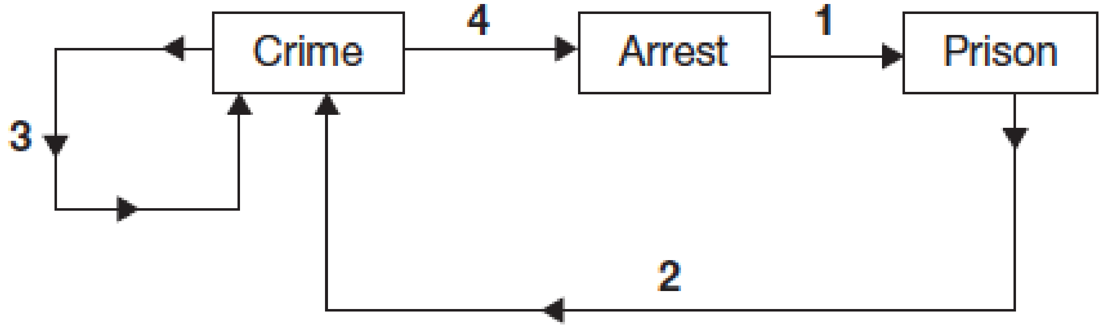
$$P_{R2}(1 - P_I)\ P_A$$
$$+ P_{R2}(1 - P_I)\ P_{R1}(1 - P_A)\ P_A$$
$$+ P_{R2}(1 - P_I)\ [\ P_{R1}(1 - P_A)\ ]^2\ P_A \ldots$$
$$+ P_{R2}(1 - P_I)\ [\ P_{R1}(1 - P_A)\ ]^k\ P_A + \ldots$$

GEOMETRIC SERIES

FIRST TERM: $P_{R2}\ (1 - P_I)\ P_A$

RATIO: $P_{R1}\ (1 - P_A)$

SUM IS $$\frac{P_{R2}\ (1 - P_I)\ P_A}{1 - P_{R1}\ (1 - P_A)}$$



B

Arrest □□□Prison □□□ Crime □□□ Arrest

Arrest □□□Prison □□□ Crime □□□ Crime □□□ Arrest

Arrest □□□Prison □□□ Crime □□□ Crime □□□ Crime □□□ Arrest

...

$P_I \, P_{R3} P_A$

$+ \; P_I \, P_{R3} \, [\, P_{R1}(1 - P_A) \,] \, P_A$

$+ \; P_I \, P_{R3} \, [P_{R1}(1 - P_A)]^2 \, P_A +$ ...

$+ \, P_I \, P_{R3} \, [P_{R1}(1 - P_A)]^k \, P_A +$ ..

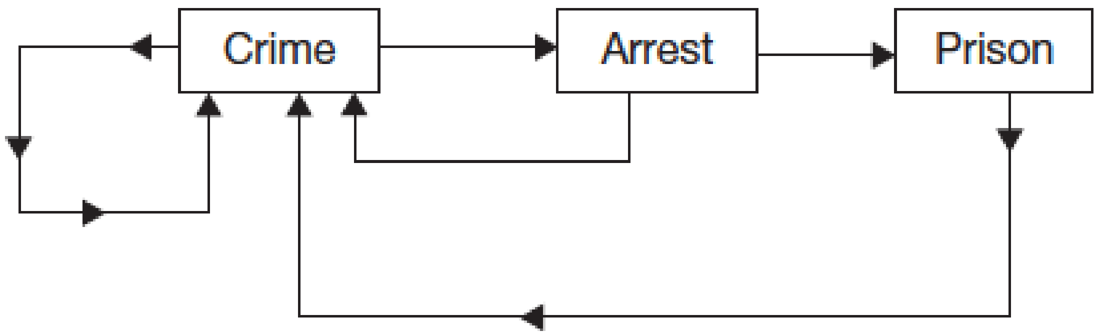
$$= \frac{P_I P_{R3} P_A}{1 - P_{R1}(1 - P_A)}$$

A + B gives:

Pr(A | A) =

$$\frac{P_{R2}(1 - P_I) P_A}{1 - P_{R1}(1 - P_A)} + \frac{P_I P_{R3} P_A}{1 - P_{R1}(1 - P_A)}$$

Pr(I | I)



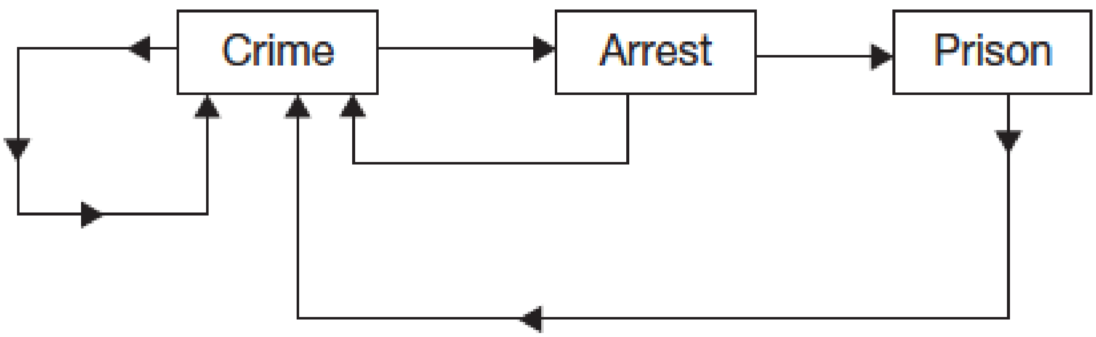
1 Crime

Prison □ Crime □ Arrest □ Prison

2 Crimes

Prison □□□Crime □□□Crime □□□Arrest □□□Prison

Prison □□□Crime □□□Arrest □□□Crime □□□Arrest □□Prison



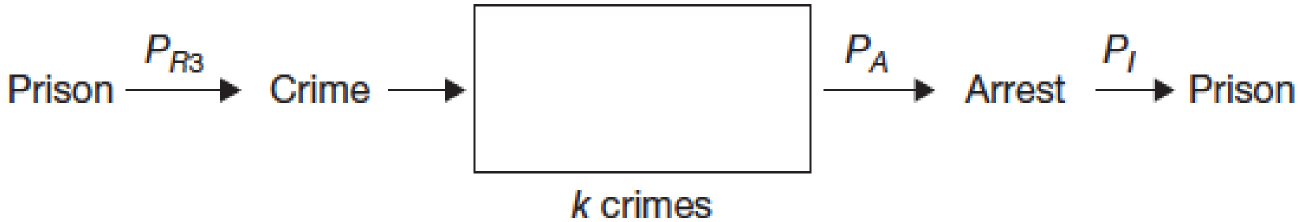
3 Crimes

Prison □□3□Crimes □□□Arrest □□□Prison

Prison □□□2□Crimes □□□Arrest
□□□Crime □□□Arrest □□□Prison

Prison □□□Crime □□□Arrest
□□2□□Crimes □□□Arrest □□□Prison

Prison □□□Crime □□□Arrest □□□Crime
□□□Arrest □□□Crime □□□Arrest
□□□Prison



j (no arrest)	*k*-*j* (arrest)
$(1 - P_A)\, P_{R1}$	$P_A\, (1 - P_I)\, P_{R2}$

Consider any sequence of *k* crimes involving ($k - j$) arrests.

The probability of such a sequence is:

$$[\,(1 - P_A)\, P_{R1}\,]^j \quad [\,P_A\,(1 - P_I)\, P_{R2}\,]^{k-j}$$

$$x^j \qquad\qquad y^{k-j}$$

How many such sequences
(for a fixed *k* and *j*)?

Number of ways of choosing *j* slots in *k* places =

$$\binom{k}{j}$$

Pr(I | I) for *k* crimes = $P_{R3}\, P_A\, P_I$

$$\sum_{j=0}^{k} \binom{k}{j} x^j \, y^{k-j} = P_{R3}\, P_A\, P_I \,(x + y)^k$$

Now Pr(I | I) =

$$P_{R3}\, P_A\, P_I \sum_{k=0}^{\infty} (x + y)^k$$

$$\Pr(I \mid I) =$$

$$P_{R3}\, P_A\, P_I \sum_{k=0}^{\infty} (x + y)^k$$

$$= \frac{P_{R3}\, P_A\, P_I}{1 - (x+y)}$$

$$=$$

$$\frac{P_{R3}\, P_A\, P_I}{1 - [(1 - P_A)\, P_{R1} + P_A\, (1 - P_I)\, P_{R2}\,]}$$

Comparison of The Three Measures

Simplifying Assumptions:

1) The criminal justice system has no effect on an individual's propensity to return to crime:

$$P_{R1} = P_{R2} = P_{R3} = P$$

Then Pr(C | C) = P

$$Pr(A \mid A) = \frac{P\,P_A}{1 - Pr(1 - P_A)}$$

$$Pr(I \mid I) = \frac{P\,P_A P_I}{1 - P + P_A P_I P}$$

2) $PA = PI = 1/4$

These assumptions yield

$$Pr(C \mid C) = P$$
$$Pr(A \mid A) = P/(4 - 3P)$$
$$Pr(I \mid I) = P/(16 - 15P)$$

so the transition matrix looks like

$$\begin{matrix} \text{No Recidivism} \\ \text{Crime} \\ \text{Arrest} \\ \text{Prison} \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{3(1 - P)}{4} & \frac{3P}{4} & \frac{1}{4} & 0 \\ \frac{3(1 - P)}{4} & \frac{3P}{4} & 0 & \frac{1}{4} \\ 1 - P & P & 0 & 0 \end{pmatrix}$$

The fundamental matrix $N = (I - Q)^{-1}$ is

$$N = \frac{1}{1 = P} \begin{array}{c} \\ \text{Crime} \\ \text{Arrest} \\ \text{Prison} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Crime} & \text{Arrest} & \text{Prison} \end{array} \\ \begin{pmatrix} 1 & 1/4 & 1/16 \\ P & (4 - 3P)/4 & (4 - 3P)/16 \\ P & P/4 & (16 - 15P)/16 \end{pmatrix} \end{array}$$

$$\Pr(C \mid C) = P_{R1}(1 - P_A) + P_A P_{R2}(1 - P_I) + P_A P_I P_{R3}$$

$$If\ \ P_{R1} = P_{R1} = P_{R1} = p$$

$$Pr(C \mid C) = p(1 - P_A) + P_A\, p\,(1 - P_I) + P_A P_I\, p$$

$$= p\,[\,1 - P_A) + P_A(1 - P_I) + P_A P_I]$$

$$= p$$

$$Pr(A \mid A) \quad =$$

$$\frac{P_A \left[P_{R2} \; (1 - P_I) \; + P_I \, P_{R3} \right]}{1 - P_{R1} \, (1 - P_A)}$$

$$= \frac{P_A \left[p \; (1 - P_I) \; + P_I \, p \right]}{1 - p(1 - P_A)}$$

$$= \frac{pP_A}{1 - p \, (1 - P_A)}$$

$$Pr(I \mid I) = \frac{P_{R3}\, P_A\, P_I}{1 - P_{R1}(1 - P_A) - P_A(1 - P_I)\, P_{R2}}$$

$$= \frac{p\, P_A\, P_I}{1 - p(1 - P_A) - P_A(1 - P_I)\, p}$$

$$= \frac{p\, P_A\, P_I}{1 - p + pP_A\, P_I}$$

***Second Assumption:* PA = PI = 1/4**

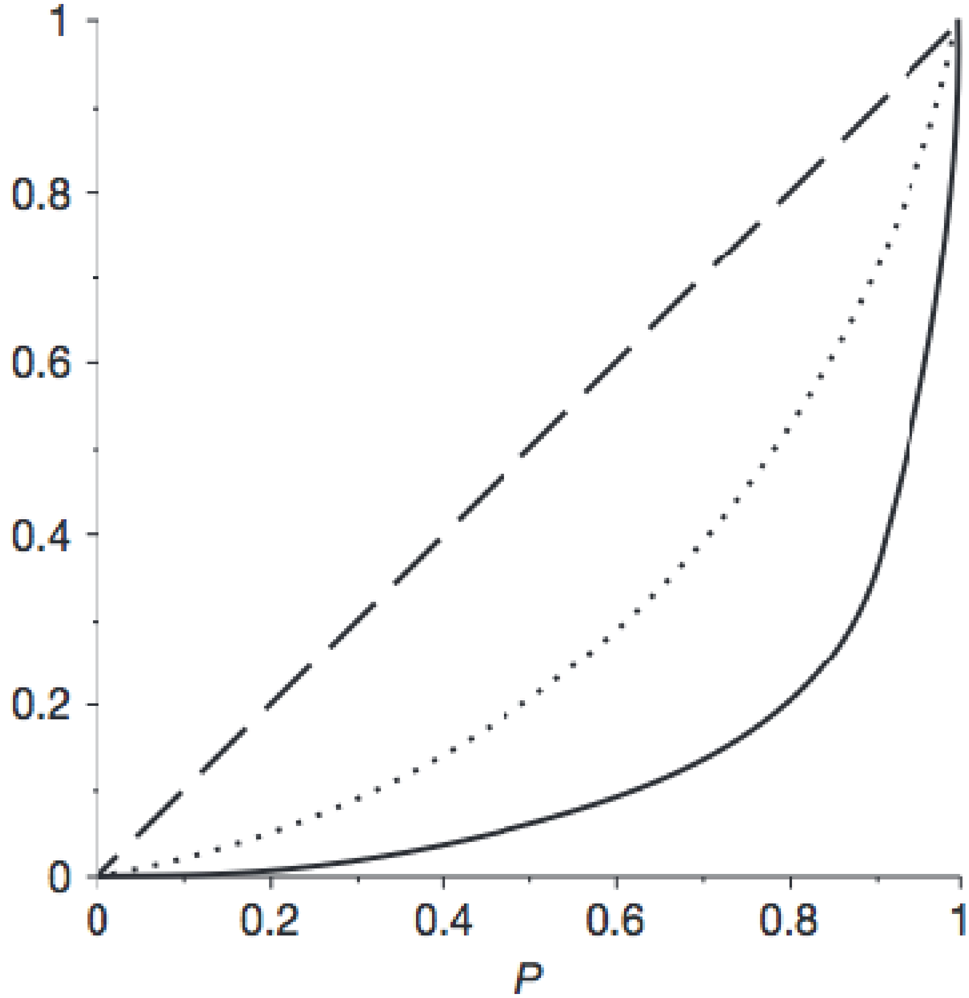
$$Pr(C \mid C) = p$$

$$Pr(A \mid A) = \frac{pPA}{1 - p(1 - PA)} = \frac{\frac{p}{4}}{1 - \frac{3p}{4}} =$$

$$\frac{p}{4 - 3p}$$

$$Pr(I \mid I) = = \frac{p\,PA\,PI}{1 - p + pPA\,PI} =$$

$$\frac{p\frac{1}{4}\frac{1}{4}}{1-p+\frac{1}{4}\frac{1}{4}p} = \frac{p}{16 - 15p}$$



$$Pr(C \mid C) = p$$

$$Pr(A \mid A) = \frac{p}{4 - 3p}$$

$$Pr(I \mid I) = \frac{p}{16 - 15p}$$

Example:

$p = .9$ (high rate of crime repetition)

Then $Pr(A \mid A) = \dfrac{p}{4 - 3p} = .69$

$Pr(I \mid I) = \dfrac{p}{16 - 15p} = .36$

(low rate of reincarceration)

Going in the opposite direction:

Suppose $Pr(I \mid I) = 1/10$

Then $$\frac{p}{16 - 15p} = \frac{1}{10}$$

so $p = 16/25 = .64 = Pr(C \mid C)$

and $Pr(A \mid A)$

$$= \frac{\frac{16}{25}}{4 - \frac{48}{25}} = \frac{16}{52} \sim .31$$

$$tCrime = \frac{1 + \frac{1}{4} + \frac{1}{16}}{1 - p}$$

$$tArrest = \frac{p + \frac{4-3p}{4} + \frac{4-3p}{16}}{1-p} = \frac{p + 20}{16\,(1 - p)}$$

$$tIncarceration = \frac{p + \frac{p}{4} + \frac{16 - 15p}{16}}{1 - p} =$$

$$\frac{5p + 16}{16\,(1 - p)}$$

$$t = \frac{1}{16(1-p)} \begin{pmatrix} 21 \\ 20 + p \\ 16 + 5p \end{pmatrix}$$

With $p = 9/10$,

$$N = \begin{array}{c} \\ \text{Crime} \\ \text{Arrest} \\ \text{Prison} \end{array} \begin{array}{c} \text{Crime Arrest Prison} \\ \begin{pmatrix} 10 & 5/2 & 5/8 \\ 9 & 13/14 & 13/16 \\ 9 & 9/4 & 25/16 \end{pmatrix} \end{array}$$

We can't observe:

$N_{CC}, N_{CA}, N_{CI}, N_{AC}, N_{IC}$

But we can observe:

$N_{AA}, N_{AI}, N_{IC}, N_{II}$

BLUMSTEIN-LARSON

The number of subsequent arrests on index charges of a person arrested for a first index change ranged from 2.2 to 2.8:

Now $N_{AA} = \frac{4 - 3p}{4 - 4p}$ so with $x = \frac{4 - 3p}{4 - 4p}$, we have $p = \frac{x - 1}{x - 3/4}$

$$t = \frac{1}{16(1-p)} \begin{pmatrix} 21 \\ 20 + p \\ 16 + 5p \end{pmatrix}$$

With $p = 9/10$

$$t = \begin{pmatrix} 210/16 \\ 209/16 \\ 205/16 \end{pmatrix} \sim \begin{pmatrix} 13.125 \\ 13.06 \\ 12.81 \end{pmatrix}$$

x	$\frac{x-1}{x-3/4}$
1	0
3/2	.67
2	.8
5/2	.86
3	.89
3.2	.9
7/2	.91
3.8	.92
4	.92
5	.94
6	.95
7	.96
8	.97

For range [3.2, 3.8], p varies between .9 and .92

With p =.91:

$N_{AI} = .89$ $N_{IA} = .2.53$

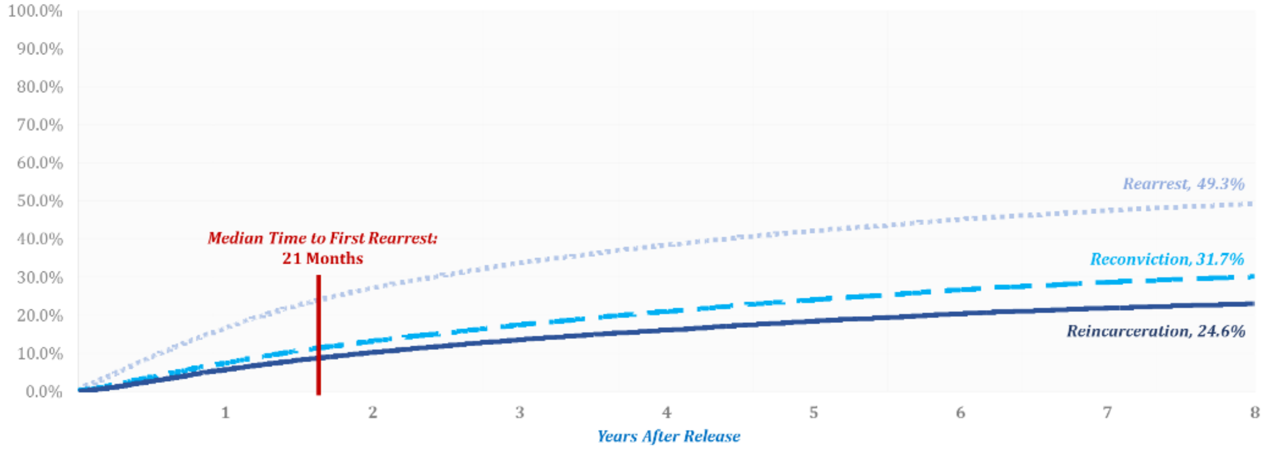
$N_{II} = 1.67$ $N_{CC} = 11$

The Commission found that two

factors – offenders' criminal histories and their ages at the time of release into the community – were most closely associated with differences in recidivism rates. Younger offenders recidivated at significantly higher rates than older offenders, and offenders with more extensive criminal histories recidivated at significantly higher rates than offenders with lesser criminal histories.

Figure 4.
Time to First Rearrest of Recidivism Study Offenders

SOURCE: U.S. Sentencing Commission's 2005 Recidivism Release Cohort Datafile, RECID05.



SOURCE: U.S. Sentencing Commission's 2005 Recidivism Release Cohort Datafile, RECID05. The reconviction and reconfinement lines indicate time to first arrest that led to a conviction and time to first arrest that led to a confinement, respectively.

Table 2.
Rearrest Rates for Recidivism Study Offenders

Years After Release	%	Cumulative %
One Year After Release	16.6%	16.6%
Two Years After Release	10.5%	27.1%
Three Years After Release	6.6%	33.7%
Four Years After Release	4.7%	38.4%
Five Years After Release	3.7%	42.1%
Six Years After Release	3.0%	45.1%
Seven Years After Release	2.3%	47.5%
Eight Years After Release	1.8%	49.3%

SOURCE: U.S. Sentencing Commission's 2005 Recidivism Release Cohort Datafile, RECID05.