Olinick, *Mathematical Modeling for the Social and Life Sciences*: References

CHAPTER 1 MATHEMATICAL MODELS

Adair, Robert, *The Physics of Baseball*, New York: Harper and Row, 1990. (For a number of applications where the initial velocity is decidedly nonzero!)

Azevedo, Jane, *Mapping Reality: An Evolutionary Realist Methodology for the Natural and Social Sciences*, Albany: State University of New York Press, 1997.

Borges, Jorge Luis, "On Exactitude in Science," *Collected Fictions*, Andrew Hurley, trans., New York: Penguin Books, 1998.

Boyce, William E., and Richard C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, 9th ed., New York: Wiley, 2009. (See especially Exercise 27 on page 65 for a model of a body falling in a relatively dense fluid that made possible the determination of the charge on an electron.)

Drake, Stillman, "Newton's Apple and Galileo's Dialogue," *Scientific American* 243(2) (August, 1980): 151–156.

Drake, Stillman, "Galileo's Discovery of the Law of Free Fall." *Scientific American* 228(5) (May 1973): 84–92.

Drake, Stillman, "The Role of Music in Galileo's Experiments." *Scientific American* 232(6) (June 1975): 98–104.

Drake, Stillman, *Galileo at Work*, Chicago: University of Chicago Press, 1978.

Drake, Stillman, *Galileo*: *Pioneer Scientist*, Toronto: University of Toronto Press, 1990.

Edwards, Dilwyn and Mike Hanson, *Guide to Mathematical Modeling*, Boca Raton, Florida: CRC Press, 1990. (For models of free fall in which air resistance, drag, and parachutes are considered.)

Gaddis, John Lewis, *The Landscape of History*: *How Historians Map the Past*, Oxford: Oxford University Press, 2002.

Judson, Horace Freeland, *The Search for Solutions*, New York: Holt, Rinehart and Winston, 1980.

Kemeny, John G., and J. Laurie Snell, "On the Methodology of Mathematical Models," in *Mathematical Models in the Social Sciences*, Boston: Ginn, 1962.

Kline, Morris, *Mathematics and the Physical World*, New York: Crowell, 1959.

Maki, Daniel P., and Maynard Thompson, Chapter I of *Mathematical Models and Applications*, Englewood Cliffs, N.J.: Prentice-Hall, 1973.

McNeil, Maureen, "Newton as National Hero," in John Fauvel et al., eds., *Let Newton Be!* Oxford: Oxford University Press, 1988. (For an interesting discussion of the cultural significance of the falling apple myth.)

Morris, Richard, *Time*'s *Arrows*: *Scientific Attitudes toward Time*, New York: Simon and Schuster, 1984.

Simmons, George F., *Differential Equations with Applications and Historical Notes*, New York: McGraw-Hill, 1991.

Westfall, Richard S., *Never at Rest: A Biography of Isaac Newton*, Cambridge: Cambridge University Press, 1983. (See page 154 for the origin of the apple story.)

CHAPTER 2 STABLE AND UNSTABLE ARMS RACES

The development of the model of an arms race discussed in this chapter is due to L. F. Richardson, "Generalized Foreign Policy," *British Journal of Psychology Monographs Supplements* 23 (1939). Richardson's more extended writings on the causes of war may be found in his two books: *Statistics of Deadly Quarrels*, Chicago: Quadrangle Books, 1960a; and *Arms and Insecurity: A Mathematical Study of the Causes and Origins of War*, Pittsburgh: Boxwood Press, 1960b. The Preface to *Arms and Insecurity* was written by Nicolas Rashevsky and Emesto Trucco.

Richardson's assembled writings are contained in two hefty volumes, *Collected Papers of Lewis Fry Richardson*, Vol. 1: *Meteorology and Numerical Analysis*; Vol. 2: *Quantitative Psychology and Studies of Conflict*, Cambridge: Cambridge University Press, 1993.

The treatment of the Richardson model presented here derives from, and was inspired by, chapters 1 and 2 of Anatol

Rapoport's *Fights, Games and Debates*, Ann Arbor: University of Michigan Press, 1960. For further reading in this area, see Thomas L. Saaty, *Mathematical Models of Arms Control and Disarmament*, New York: Wiley, 1968.

A meteorologist and close friend of Richardson, Oliver M. Ashford, wrote the first full-length biography of Richardson: *Prophet or Professor? The Life and Work of Lewis Fry Richardson*, Bristol, England: Adam Hilger, 1985. The mathematics underlying some of Richardson's research is developed and integrated with a history and description of that work in Thomas W. Körner's delightful book *The Pleasures of Counting*, Cambridge: Cambridge University Press, 1996; see chapter 8, "A Quaker Mathematician," and chapter 9, "Richardson on War."

General mathematical discussions of systems of linear differential equations with constant coefficients may be found in William E. Boyce and Richard C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, 9th ed., New York: Wiley, 2010.

The use of derivatives to approximate function values is discussed in Earl Swokowski, Michael Olinick, and Dennis Pence, *Calculus*, 6th ed., Boston: PWS, 1994.

Richard L. Burden and J. Douglas Faires discuss the numerical solution of differential equations at length in *Numerical Analysis*, 9th ed., Pacific Grove, CA: Brooks/Cole, 2011.

Richardson's studies of coastline measurements appeared as "The Problem of Contiguity: an Appendix of Statistics of Deadly Quarrels," *General Systems Yearbook* 6 (1961): 139–187. Mandelbrot's initial use of these studies can be found in "How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension," *Science* 155 (1967): 636–638. A more extended treatment with his introduction to the concept of fractals appears as chapter 5 in Mandelbrot's major book *The Fractal Geometry of Nature*, New York: Freeman, 1977.

Bateson's writings are found in Gregory Bateson, *Steps to an Ecology of Mind*, Scranton: Chandler Publishing Company, 1972.

Eckhardt's assessment is contained in William Eckhardt, *Civilizations*, *Empires and Wars*: *A Quantitative History of War*, Afterword by David Wilkinson, Jefferson NC: McFarland, 1992.

Reviews of the arms race literature may be found in Craig Etcheson, *Arms Race Theory*: *Strategy and Structure of Behavior*, Westport CT: Greenwood, 1989; Michael Intriligator and Dagobert Brito, "Richardsonian Arms Race Models," in *Handbook of War Studies*, Mans Midlarsky, ed.; Ann Arbor: University of Michigan Press, 1989; and Walter Isard and Charles Anderton, "A Survey of Arms Race Models," in *Arms Races, Arms Control, and Conflict Analysis*, Walter Isard, ed., Cambridge: Cambridge University Press, 1988. Richardson's model, its extensions, and testing are thoroughly presented in a wider context in Todd Sander and Keith Hartley, *The Economics of Defense*, Cambridge: Cambridge University Press, 1995.

CHAPTER 3 ECOLOGICAL MODELS: SINGLE SPECIES

Population models for human and nonhuman organisms are developed in detail in each of the following books:

Emmell, Thomas C., *An Introduction to Ecology and Population Biology*, New York: Norton, 1973.

Grossman, Stanley I., and James E. Turner, *Mathematics for the Biological Sciences*, New York: Macmillan, 1974.

Pie1ou, E. C., *An Introduction to Mathematical Ecology*, New York: Wiley, 1969.

Poole, Robert W., *An Introduction to Quantitative Ecology*, New York: McGraw-Hill, 1974.

More sophisticated models of human population growth can be found in the following:

Harrison, G. A., and A. J. Boyce, eds., *The Structure of Human Populations*, Oxford: Clarendon Press, 1972.

Pollard, J. H., *Mathematical Models for the Growth of Human Populations*, Cambridge: Cambridge University Press, 1973.

Ralph Thomlinson presents a survey of world population trends and population theories in his book *Population Dynamics: Causes and Consequences of World Demographic Change*, New York: Random House, 1965.

The use of exponential decay models to date ancient objects is presented in W. F. Libby, *Radiocarbon Dating*, Chicago: University of Chicago Press, 1955; and Henry Faull, *Ages of Rocks, Planets and Stars*, New York: McGraw-Hill, 1966.

The sad fate of the peregrine falcon is reported in the papers of a research conference edited by Joseph J. Hickey, *Peregrine Falcon Populations: Their Biology and Decline*, Madison: University of Wisconsin Press, 1969.

Pearl and Reed's original paper on a logistic model for U.S. population is "On the Rate of Growth of the United States Population since 1790 and Its Mathematical Representation," *Proceedings of the National Academy of Sciences* 6 (1920): 275–288. Pearl gives an extended discussion of this work in his book *Studies in Human Biology*, Baltimore: William and Wilkins, 1924. A critical history of the logistic model that contains complete references to the works of Quetelet and Verhulst is G. Udny Yule's paper, "The Growth of Population and the Factors Which Control It," *Journal of the Royal Statistical Society* 88 (1925): 1–58. The more recent cited paper by D. O. Cogwill is "The Use of the Logistic Curve and the Transition Model in Developing Nations," in Bose, Desai, and Rain, eds., *Studies in Demography*, Chapel Hill: University of North Carolina Press, 1970.

The first popular treatment of chaos theory is James Gleick's book *Chaos*: *Making a New Science*, New York: Viking, 1987. Trinh Xuan Thuan's account is contained in *Chaos and Harmony*: *Perspectives on Scientific Revolutions of the Twentieth Century*, Oxford: Oxford University Press, 2001. More extended mathematical treatments of chaos in the discrete logistic model can be found in Fred Brauer and Carlos Castillo-Châvez, *Mathematical Models in Population Biology and Epidemiology*, New York: Springer, 2001. James T. Sandefur, *Discrete Dynamical Systems: Theory and Applications*, Oxford: Clarendon Press, 1990, and Steven H. Strogatz, *Nonlinear Dynamics and Chaos*, Cambridge: West View Press, 1994. An excellent introduction to chaos theory for students of calculus is David Acheson, *From Calculus to Chaos: An Introduction to Dynamics*, Oxford: Oxford University Press, 1997.

The works of Colin W. Clark mentioned in Suggested Project 3 include "Economically Optimal Policies for the Utilization of Biologically Renewable Resources," *Mathematical Biosciences* 12 (1971): 245–260; and "The Dynamics of Commercially Exploited Natural Animal Populations," *Mathematical Biosciences* 13 (1972): 149–164.

Allee's main accounts of his work are Warder Clyde Allee, *Animal Aggregations: A Study in General Sociology*, Chicago: University of Chicago Press, 1931; *The Social Life of Animals*, London: Heinneman; and *Principles of Animal Ecology*, Philadelphia: W. B. Saunders, 1949. For more about the Allee effect in biology, see Franck Courchamp, Ludec Berce, and Joanna Gascoigne, *Allee Effects in Ecology and Conservation*, Oxford: Oxford University Press, 2008. Saber Elaydi and Robert Sacker provide some discrete models of population growth with the Allee effect in their paper "Population Models with Allee Effects: A New Model," *Journal of Biological Dynamics* 4 (2010): 397–408. For a more extended treatment of population dynamics, see Peter Turchin, *Complex Population Dynamics: A Theoretical/Empirical Synthesis*, Princeton: Princeton University Press, 2003.

CHAPTER 4 ECOLOGICAL MODELS: INTERACTING SPECIES

Mathematical Ecology

The texts by Evelyn Pielou and Robert Poole cited in the References of Chapter 3 are excellent introductions to the broad area of mathematical ecology. Lotka's pioneering work is available in his book *Elements of Mathematical Biology*, New York: Dover, 1956. A useful survey of nonlinear models of population growth is given by Narendra Goel, Samaresh Maitra, and Elliott Montroll in *On the Volterra and Other Nonlinear Models of Interacting Populations*, New York: Academic Press, 1971.

Lawrence B. Slobodkin (1928–2009) was an ecologist who warned against the uncritical acceptance of mathematical models by biologists. See his book *Growth and Regulation in Animal Populations*, New York: Holt, Rinehart and Winston, 1961, 2nd enl. ed., Dover Press, 1980, as well as "Comments from a Biologist to a Mathematician," in Simon A. Levin, ed., *Ecosystem Analysis and Prediction*, Philadelphia: Society for Industrial and Applied Mathematics, 1975, 318–329. He begins the paper with a list of "ten things I very much wish mathematicians would stop doing in population biology." *A Citizen's Guide to Ecology*, Oxford: Oxford University Press, 2003.

Exercise 30 is adapted from John G. Kemeny and J. Laurie Snell, *Mathematical Models in the Social Sciences*, Boston: Ginn, 1962, 33.

Competition

Mayr's comments on competition may be found in his book *Populations*, *Species and Evolution*, Cambridge: Harvard University Press, 1970. For an earlier approach, see G. F. Gause, *The Struggle for Existence*, New York: Hafner, 1934. Pielou and Poole both describe some of the experimental tests of the model.

For exceptions to the competitive exclusion principle and explanations of why they exist in nature, see Michael Huston, *Biological Diversity: The Coexistence of Species on Changing Landscapes*, Cambridge: Cambridge University Press, 1993; and M. Huston, D. DeAngelis, and W. Post, "New Computer Models Unify Ecological Theory," *Bioscience* 38(10) (1988): 682–691.

Tom Ray's discussion may be found in "An Evolutionary Approach to Synthetic Biology: Zen and the Art of Creating Life," *Artificial Life* 1 (1994): 179–209.

Predation

In his *Animal Ecology*, New York: McGraw-Hill, 1931, Royal N. Chapman presents an English translation of some of Volterra's work under the title "Variations and Fluctuations of the Number of Individuals in Animal Species Living Together." Also of interest are Volterra, *Leçons sur la théorie mathématique de lutte pour la vie*, Paris: Gauthier-Villars, 1931, 1990; and D'Ancona, U., *The Struggle for Existence*, Leiden: Brill, 1954.

Huffaker's study "Experimental studies on predation: dispersion factors and predator-prey oscillations" has been reprinted in *Readings in Population and Community Biology*, W. E. Hazen, ed., Philadelphia: Saunders, 1970.

Leigh's paper, "The ecological role of Volterra's equations," appears in *Lectures on Mathematics in the Life Sciences: Some Mathematical Problems in Biology*, M. Gerstenhaber, ed., Providence, R.I.: American Mathematical Society, 1968.

The graphical results of the Leslie-Gower model have been reprinted from Pielou's book, which contains a more complete discussion. See also P. H. Leslie and J. C. Gower, "The Properties of a Stochastic Model for the Predator-Prey Type of Interaction between Two Species," *Biometrika* 46 (1960): 219–234.

The Harrison and Rosenzweig-MacArthur models are found in Gary Harrison, "Comparing Predator-Prey Models to Luckinbill's Experiment with *Didinium* and *Paramecium*," *Ecology* 76 (1995): 357–374; and M. L. Rosenzweig and R. H. MacArthur, "Graphical Representation and Stability Conditions of Predator-Prey Interactions," *American Naturalist* 97 (1963): 209–223. See also Leo S. Luckinbill, "Coexistence in Laboratory Populations of *Paramecium Aurelia* and Its Predator *Didinium Nasutum*," *Ecology* 54 (1973): 1320–1327.

Werner Krebs studies an approach for predation in an ecosystem with multiple species in "A General Predator-Prey Model," *Mathematical and Computer Modeling of Dynamical Systems* 9 (2003): 387–401.

Lotka-Volterra in other Contexts

George Bell's work is described in his paper, "Predator-Prey Equations Simulating an Immune Response," *Mathematical Biosciences* 16 (1973): 291–314.

Richard M. Goodwin's original model appears in his paper "A Growth Cycle," in Feinstein, C. H., ed., *Socialism, Capitalism and Economic Growth*, Cambridge: Cambridge University Press, 1967. For a discussion (and appreciation) of Goodwin's predator-prey model and subsequent developments, see Roberto Veneziani and Simon Mohun, "Structural Stability and Goodwin's Growth Cycle," *Structural Change and Economic Dynamics* 17 (2006): 437–451.

Joshua Epstein introduces analogies between Lotka-Volterra models in ecology, epidemiology, and political science in his book *Nonlinear Dynamics, Mathematical Biology, and Social Science*, Santa Fe Institute Studies in the Sciences of Complexity, Vol. IV (Reading, MA: Addison-Wesley, 1997). Epstein provides the quotation from Harry Olson, *Dynamical Analogies*, 2nd ed., Princeton: Van Nostrand, 1958, in a more extended treatment of analogy or "mathematical metaphor."

For applications of the Lotka-Volterra model to vampirology, see Richard F. Hartl, Alexander Mehlmann, and Andreas Novak, "Cycles of Fear: Periodic Bloodsucking Rates for Vampires," *Journal of Optimization Theory and Applications* 75 (1992): 559–568. This paper extends earlier studies by Hartl and Mehlmann: "The Transylvanian Problem of Renewable Resources," *Révue Francaise d'Automatique, Informaique et de Recherche Operationelle* 16 (1982): 379–390; and "Convex-Concave Utility Function: Optimal Blood-Consumptions for Vampires," *Applied Mathematical Modelling* 7 (1983): 83–88. Optimal behavior for the humans is discussed in Dennis J. Snower, "Macroeconomic Policy and the Optimal Destruction of Vampires," *The Journal of Political Economy* 90 (1982): 647–655.

P. Jeffrey Brantingham et al., "The Ecology of Gang Territorial Boundaries," *Criminology* 30, 851–885, employs the Lotka-Volterra system to examine territory formation among urban street gangs. Their research shows that the most dangerous place to be in a neighborhood packed with gangs is not deep within the territory of a specific gang but on the border between two rival groups. "It's no coincidence that Lotka-Volterra equations would have bearing on the configurations of gang territories: The same forces that define territories in the animal kingdom also are at work in all kinds of rivalries between groups of people" they emphasize.

Mathematical Background

In addition to the texts on differential equations cited earlier and Seeley's book, see David A. Sanchez, *Ordinary Differential Equations and Stability Theory*: *An Introduction*, San Francisco: Freeman, 1968; New York: Dover, 1979, Chapter 4.

Note also the following:

John H. Hubbard and Beverly H. West, *Differential Equations*: *A Dynamical Systems Approach, Ordinary Differential Equations*, New York: Springer, 1997.

Morris W. Hirsch, Stephen Smale, and Robert L. Devaney, *Differential Equations, Dynamical Systems and an Introduction to Chaos*, Amsterdam: Elsevier, 2004.

For an extended discussion (with many examples) of linearizing systems of differential equations near equilibrium points, see Paul Blanchard, Robert L. Devaney, and Glen R. Hall, *Differential Equations*, Belmont, CA: Thompson Brooks/Cole, 2006. You can find a complete treatment of the theory, with proofs, in Jack Hale and Hüseyin Koçak, *Dynamics and Bifurcations*, New York: Springer, 1991.

CHAPTER 5 TUMOR GROWTH MODELS

L. von Bertalanffy, "Quantitative Laws in Metabolism and Growth," *Quarterly Review of Biology* 32 (1957): 217–231.

Mark Davidson, *Uncommon Sense: The Life and Thought of Ludwig von Bertalanffy, Father of General Systems Theory*, Los Angeles: J.P. Tarcher, 1983.

E. B. Cox, M. A. Woodbury, L. E. Myers, "A New Model for Tumor Growth Analysis Based on a Postulated Inhibitory Substance," *Comp. Biomed. Res.* 13 (1980): 437.

J. P. Freyer, "Role of Necrosis in Regulating the Growth Saturation of Multicellular Spheroids," *Cancer Research* 48 (1988): 2432–2439.

J. P. Freyer, R. M. Sutherland, "Regulation of Growth Saturation and Development of Necrosis by the Glucose and Oxygen Supply," *Cancer Research* 46 (1986): 3513–3520.

Benjanin Gompertz, "On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies," *Philosophical Transactions of the Royal Society of London* 36 (1825): 513.

David Guterson, *East of the Mountains*, New York: Harcourt Brace, 1999.

Gloria Jarne, Julio Sánchez-Chóliz and Francisco Fátas-Villafranca, "'S-Shaped' Economic Dynamics: The Logistic and Gompertz Curves Generalized," *Electronic Journal of Evolutionary Modeling and Economic Dynamics* (www.e-jemed.org), Article number 1048, 2005. See also "'S-shaped' Curves in Economic Growth: A Theoretical Contribution and an Application," *Evolutionary and Institutional Economics Review* 3(2) (2007): 239–259.

Matthew D. Johnston, "Mathematical Modelling of Cell Population Dynamics in the Colonic Crypt with Application to Colorectal Cancer," Ph.D. thesis, University of Oxford, December 2008.

Matthew D. Johnston et al., "Examples of Mathematical Modeling: Tales from the Crypt." *Cell Cycle* 6(17) (2007): 2106–2112.

Matthew D. Johnston, Carina M. Edwards, Walter F. Bodmer, Philip K. Maini, and S. Jonathan Chapman, "Mathematical Modeling of Cell Population Dynamics in the Colonic Crypt and in Colorectal Cancer," *Proceedings of the National Academy of Sciences of the United States of America* 104(10) (2006): 4008–4013.

Anna Kane Laird, "Dynamics of Tumor Growth," *British Journal of Cancer* 18 (1964): 490–502.

Anna Kane Laird, "Dynamics of Tumour Growth: Comparison of Growth Rates and Extrapolation of Growth Curve to One Cell," *British Journal of Cancer*, 19(2) (1965): 278–291.

Anna Kane Laird, Sylvanus A. Tyler, and Ambrose D. Barton, "Dynamics of Normal Growth," *Growth* 29 (1965): 323–348.

J. Landry, J. P. Freyer, R. M. Sutherland, "A Model for the Growth of Multicellular Spheroids," *Cell and Tissue Kinetics* 15 (1982): 585–594.

M. Marušić, Z. Bajzer, "Generalized two-parameter equation of growth," *Journal of Mathematical Analysis and Applications* 179 (1993): 446–462.

M. Marušić, Z. Bajzer, J. P. Freyer, S. Vuk-Pavlovic, "Modeling Autostimulation of Growth in Multicellular Tumor Spheroids," *International Journal of Bio-medical Computing* 29 (1991): 149–158.

M. Marušić, Z. Bajzer, J. P. Freyer, S. Vuk-Pavlovic, "Analysis of Growth of Multicellular Tumour Spheroids by Mathematical Models," *Cell Proliferation* 27 (1994): 73–94.

M. Marušić, S. Vuk-Pavlovic, "Prediction Power of Mathematical Models for Tumor Growth," *Journal of Biological Systems* 1 (1993): 69–78.

M. Marušić, "Mathematical Models of Tumor Growth," *Mathematical Communications* (Department of Mathematics, University of Osijek) 1 (1996): 175–192.

S. Piantadosi, "A Model of Growth with First-Order Birth and Death Rates," *Comp. Biomed. Res.* 18 (1985): 220–232.

M. A. Savageau, "Allometric Morphogenesis of Complex Systems: A Derivation of the Basic Equations from First Principles," *Proceedings of the National Academy of Sciences USA* 76 (1979): 6023–6025.

Daisuke Satoh, "A Discrete Gompertz Equation and a Software Reliability Growth Model," *Institute of Electronics, Information and Communication Engineers Transactions on Information and Systems*, 83-D (2000): 1508–1513.

Charles Schmidt, "The Gompertzian View: Norton Honored for Role in Establishing Cancer Treatment Approach," *Journal of the National Cancer Institute* 96 (2004): 1492–1493.

Sanjay Singh, "Projecting Teledensity in India Using Logistic and Gompertz Curve Models," *PES Business Review* 2 (1): 3–10.

Susan Sontag, *Illness as Metaphor*, Farrar, Straus and Giroux: New York, 1978.

R. M. Sutherland, "Cell and Environment Interactions in Tumor Microregions: The Multicell Spheroid Model," *Science* 240 (1988): 177.

M. E. Turner Jr., E. L. Bradley Jr., K. A. Kirk, K. M. Pruitt, "A Theory of Growth," *Mathematical Biosciences* 29 (1976): 367–373.

V. G. Vaidya, F. J. Alexandro Jr., "Evaluation of Some Mathematical Models for Tumor Growth," *International Journal of Bio-Medical Computing* 13 (1982): 19–35.

T. E. Wheldon, *Mathematical Models in Cancer Research*, Adam Hilger: Bristol, 1988.

Sewall Wright, untitled review, *Journal of the American Statistical Association* 21 (1926): 493–497.

Additional References

J. Aroesty, et al. "Tumor Growth and Chemotherapy: Mathematical Methods, Computer Simulations, and Experimental

Foundations," *Mathematical Biosciences* 17 (1973): 243–300.

R. P. Araujo and L. S. McElwain, "A History of the Study of Solid Tumour Growth: The Contributions of Mathematical Modelling," *Bulletin of Mathematical Biology* 66 (2004): 1039–1091.

L. von Bertalanffy, "Untersuchungenuber die Gesetzlichkeit des Wachstums, VII, Stoffwechseltypen und Wachstumtypen" *Biologisches Zentralblatt* 61 (1941): 510–535.

R. Bjerkvig, ed., *Spheroid Culture in Cancer Research*. CRC Press: Boca Raton, 1992.

A. D. Conger, M. C. Ziskin, "Growth of Mammalian Multicellular Tumor Spheroids," *Cancer Research* 43 (1983): 556–580.

R. D. Cook, S. Weisberg, "Linear and Nonlinear Regression," in *Statistical Methodology in the Pharmacological Sciences*, D. A Berry, ed., Marcel Dekker: New York, 1990, 163–199.

J. Durbin, G. S. Watson, "Testing for Serial Correlation in Least Squares Regression. I," *Biometrika* 37 (1950): 409–428.

J. Durbin, G. S. Watson, "Testing for Serial Correlation in Least Squares Regression. II." *Biometrika* 38 (1951): 159–177.

E. Kreyszig, *Introductory Mathematical Statistics*, Wiley: New York, 1970.

S. A. Maggelakis, J. A. Adam, "Mathematical Model of Prevascular Growth of a Spherical Carcinoma," *Mathematical and Computer Modelling* 13 (1990): 23–38.

J. J. More: "The Levenberg-Marquardt Algorithm: Implementation and Theory," in Watson, G. A., ed., *Numerical Analysis*, Springer, New York, 1977.

W. H. Press, B. P. Flannery, S. A. Teukolsky, W. T. Vetterling, *Numerical Recipes*, Cambridge University Press: Cambridge, 1986.

G. Schwarz, "Estimating the Dimension of a Model," *Annals of Statistics* 6 (1978): 461–464.

L. F. Shampine, M. K. Gordon, *Computer Solution of Ordinary Differential Equations: The Initial Value Problem*, Freeman: New York, 1975.

S. Siegel, N. J. Castellan Jr., *Nonparametric Statistics for the Behavioral Sciences*, 2nd ed., McGraw-Hill, New York, 1988.

T. E. Wheldon, J. Kirk, W. M. Grey, "Mitotic Autoregulation, Growth Control and Neoplasia," *Journal of Theoretical Biology* 38 (1973): 627.

CHAPTER 6 SOCIAL CHOICE AND VOTING PROCEDURES

For an interesting history and collection of essays of social choice theory from Pliny the Younger to Lewis Carroll, see *Classics of Social Choice*, Iain McLean and Arnold B. Urken, eds., Ann Arbor: University of Michigan Press, 1995. This volume contains translations of the important works by Borda and Condorcet. The editors provide essays by two medieval scholars who seem to have anticipated both Borda and Condorcet.

William Poundstone presents a nontechnical but entertaining account of social choice theory in his book, *Gaming the Vote*: *Why Elections Aren't Fair (and What We Can Do About It)*, New York: Hill and Wang, 2008. Poundstone presents much of the 20th-century history of the topic with a number of anecdotes about the major figures and an emphasis on the virtues of range voting.

Arrow's original paper "A Difficulty in the Concept of Social Welfare" appeared in *The Journal of Political Economy* 58(4) (August 1950): 328–346. For a fuller treatment of Arrow's approach, see his book *Social Choice and Individual Values*, 2nd ed. (New York: Wiley, 1963). The first correct proof of Arrow's Theorem may be found in J. H. Blau, "The Existence of Social Welfare Functions," *Econometrica* 25 (1957): 302–313. For an alternative proof and insight into Suggested Project 2, read P. C. Fishburn, "Arrow's Impossibility Theorem: Concise Proof and Infinite Voters," *Journal of Economic Theory* 2 (1970): 103–106.

An important early book with a mathematical perspective on social choice is Duncan Black, *The Theory of Committees and Elections*, Cambridge: Cambridge University Press, 1958. A second edition was published in 1963.

You can explore other interesting proofs of Arrow's Theorem in Salvador Barberà, "Pivotal Voters: A New Proof of Arrow's Theorem," *Economics Letters* 6 (1980): 13–16; and John Geanakpolos, "Three Brief Proofs of Arrow's Impossibility Theorem," *Economic Theory* 26 (2005): 211–215.

For a critical view of the importance of Arrow's result to social theory, see Gordon Tullock, "The General Irrelevance of the General Impossibility Theorem," *Quarterly Journal of Economics* (1967): 256–270; and Howard DeLong, *A Refutation of Arrow's Theorem*, Lanham: University Press of America, 1991.

Two recent treatments of some of the issues covered in this chapter are Wulf Gartner, *A Primer in Social Choice Theory*, Oxford: Oxford University Press, 2006; and Donald Saari, *Disposing Dictators, Demystifying Voting Paradoxes: Social Choice Analysis*, Cambridge: Cambridge University Press, 2008.

There are a number of earlier books that discuss at length the axiomatic development of the theory of social choice and voting mechanisms. These include the following:

John Bonner, *Introduction to Theory of Social Choice*, Baltimore: The Johns Hopkins University Press, 1986.

Peter C. Fishburn, *The Theory of Social Choice*, Princeton: Princeton University Press, 1973.

Jerry S. Kelly, *Arrow Impossibility Theorems*, New York: Academic Press, 1978.

Prasanta K. Paatanaik, *Voting and Collective Choice*, Cambridge: Cambridge University Press, 1971.

Amartya K. Sen, *Collective Choice and Social Welfare*, San Francisco: Holden-Day, 1970.

Alan D. Taylor, *Social Choice and the Mathematics of Manipulation*, Cambridge: Cambridge University Press, 2005.

Allan Gibbard's paper is "Manipulation of Voting Schemes: A General Result," *Econometrica* 41 (1973): 587–601; and H. P. Young's is "An Axiomatization of Borda's Rule," *Journal of Economic Theory* 9 (1974): 43–52.

Some of the discussion of the significance of Arrow's work may be found in Robert Reinhold, "Equilibrium Theorists: Kenneth J. Arrow," *The New York Times*, October 26, 1972, 71; and in two articles by Paul A. Samuelson: "Pioneers of economic thought," *The New York Times* (October 26, 1972): 71; and "The 1972 Nobel Prize for Economic Science," *Science* (November 3, 1972): 487–489.

References connected to the Gibbard-Satterthwaite result include the following:

Allan Gibbard, "Manipulation of Voting Schemes: A General Result," *Econometrica* 41 (1973): 587–601.

Mark Satterthwaite, "Strategy Proofness and Arrow's Conditions," *Journal of Economic Theory* 10 (1975): 187–217.

Alan D. Taylor, "The Manipulability of Voting Systems," *The American Mathematical Monthly* (109) (April 2002): 321–337.

Philip J. Reny, "Arrow's Theorem and the Gibbard-Satterthwaite Theorem: A Unified Approach," *Economic Letters* 70 (2001): 99–105.

E. Muller and M. A. Satterthwaite, "The Equivalence of Strong Positive Association and Strategy-Proofness," *Journal of Economic Theory* 14 (1977): 412–418.

Salvador Barberà, "Strategy-Proofness and Pivotal Voters: A Direct Proof of the Gibbard-Satterthwaite Theorem," *International Economic Review* (1983): 413–418.

Antonio Quesada, "Parallel Proofs of Arrow's and the Gibbard-Satterthwaite Theorem" (2005), http://129.3.20.41/eps/pe/papers/0504/0504003.pdf.

The example by Peter Fishburn and Steven Brams discussed in our section on Instant Runoff Voting appears in their paper "Paradoxes of Preferential Voting," *Mathematics Magazine* 56 (1983): 207–214. For a more extended argument, see their book *Approval Voting*, New York: Springer, 2007; and Brams's book *Mathematics and Democracy: Designing Better Voting and Fair-Division Procedures*, Princeton: Princeton University Press, 2008.

Some interesting papers on characterization theorems include Carlos Alós-Ferrer, "A Simple Characterization of Approval Voting," *Social Choice and Welfare* 27 (2006): 621–625; H. Peyton Young, "An Axiomatization of Borda's Rule," *Journal of Economic Theory* 9 (1974): 43–52; and B. Hansson and H. Sahlquist, "A Proof Technique for Social Choice with Variable Electorate," *Journal of Economic Theory* 13 (1976): 193–200.

Some useful references on Condorcet methods are H. Peyton Young, "Condorcet's Theory of Voting," *American Political Science Review* 82 (1988): 1231–1244; Hervé Moulin, "Condorcet's Principle Implies the No Show Paradox," *Journal of Economic Theory* 45 (1988): 53–64; and Eivind Stensholt, "Condorcet Methods—When, Why and How?" Norwegian School of Economics and Business Administration, Department of Finance and Management Science Research Paper Series No. 2008/11 (June 2008). See also Brams, *Mathematics and Democracy*.

Iain McLean, Alistair McMillan, and Burt L. Monroe, *A Mathematical Approach to Proportional Representation: Duncan Black on Lewis Carroll*, Boston: Kluwer, 1996.

For information about the Copeland Method, see Hannu Nurmi, *Comparing Voting Systems*, Dodrecht: Reidel, 1987; Donald Saari and Vincent Merlin, "The Copeland Method I: Relationships and the Dictionary," *Economic Theory* 8 (1996): 51–76; and "The Copeland Method II: Manipulation, Monotonicity, and Paradoxes," *Journal of Economic Theory* 72 (1997): 148–172. See also William Gerhlein, *Condorcet's Paradox*, Berlin: Springer, 2006.

Yann Chevaleyre, Ulle Endriss, Jérôme Lang and Nicolas Maudet, "A Short Introduction to Computational Social Choice," *Proceedings of the 33rd Conference on Current Trends in Theory and Practice of Computer Science* (SOFSEM-2007), New York: Springer-Verlag, 2007.

A study of topological choice theory might begin with some of the major papers by Graciela Chichilnisky:

"On Fixed Point Theorems and Social Choice Paradoxes," *Economics Letters* 3 (1979): 347–351.

"Social Choice and the Topology of Spaces of Preferences," *Advances in Mathematics* 37 (1980): 165–176.

"Social Aggregation Rules and Continuity," *Quarterly Journal of Economics* 97 (1982): 337–352.

"Structural Instability of Decisive Majority Rules," *Journal of Mathematical Economics* 9 (1982): 207–221.

"The Topological Equivalence of the Pareto Condition and the Existence of a Dictator," *Journal of Mathematical Economics* 9 (1982): 223–233.

Some relevant papers by Graciela Chichilnisky and Geoffrey Heal include the following:

"Necessary and Sufficient Conditions for a Resolution of the Social Choice Paradox," *Journal of Economic Theory* 31 (1983): 68–87.

"Patterns of Power: Bargaining and Incentives in Two-Person Games," *Journal of Public Economics* 23 (1984): 333–349.

"The Geometry of Implementation: A Necessary and Sufficient Condition for Straightforwardness," *Social Choice and Welfare* 14 (1997): 259–294.

Two useful works by Heal are "Social Choice and Resource Allocation: A Topological Perspective," *Social Choice and Welfare* 14 (1997): 147–160; and *Topological Social Choice*, New York: Springer, 1997.

Chichilnisky has written a compelling personal account of her life, her intellectual contributions, and her career struggles. See "Sex and the Ivy League," at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1526088.

Yuliy M. Baryshnikov's contributions include "Unifying Impossibility Theorems: A Topological Approach," *Advances In Applied Mathematics* 14 (1993): 404–415; and "Topological and Discrete Social Choice: In Search of a Theory," *Social Choice and Welfare* 14 (1997): 199–209.

See also the following works:

Luc Lauwers, "Topological Social Choice," *Mathematical Social Sciences* 40 (2000): 1–39.

Luc Lauwers, "A Note on Chichilnisky's Social Choice Paradox," *Theory and Decision* 52 (2002): 261–266.

I. D. A. Macintyre, "Two-Person and Majority Continuous Aggregation in 2-Good Space in Social Choice: A Note," *Theory and Decision* 44 (1998): 199–209.

Paras Mehta, "Topological Methods in Social Choice: An Overview," *Social Choice and Welfare* 14 (1997): 233–243.

Yasuhito Tanaka, "On the Equivalence of the Arrow Impossibility Theorem and the Brouwer Fixed Point Theorem," *Applied Mathematics and Computation* 172 (2006): 1303–1314.

Additional References on Social Choice Theory

Arrow, Kenneth J., *Collected Papers of Kenneth J. Arrow*, Cambridge: Belknap Press, 1983.

Arrow, Kenneth J., and Herve Raynaud, *Social Choice and Multicriterion Decision-Making*, Cambridge: MIT Press, 1986.

Barry, Brian, and Russell Hardin, eds., *Rational Man and Irrational Society? An Introduction and Sourcebook*, Beverly Hills, CA: Sage Publications, 1982.

Duggan, Johan, and Thomas Schwartz, "Strategic Manipulability without Resoluteness or Shared Benefits: Gibbard-Satterthwaite Generalized," *Social Choice and Welfare* 17 (2000): 85–93.

Fishburn, Peter, *Interprofile Conditions and Impossibility*, Chur, Switzerland: Harwood Academic Publishers, 1987.

Heller, Walter P., Ross M. Starr, and David A. Starrett, eds., *Social Choice and Public Decision Making*, New York: Cambridge University Press, 1986.

Mirkin, Boris G., *Group Choice*, Washington: Winston *& Sons*, 1979.

Nurmi, Hannu, *Comparing Voting Systems*, Dodrecht: Reidel, 1987.

Rapoport, Anatol, "Theory of Choice," in *Mathematical Models in the Social and Behavioral Sciences*, New York: Wiley, 1983.

Riker, William H., *Liberalism against Populism: A Confrontation between the Theory of Democracy and the Theory of Social Choice*, San Francisco: Freeman, 1982.

Saari, Donald G., *Decisions and Elections: Explaining the Unexpected*, Cambridge: Cambridge University Press, 2001.

Saari, Donald G., *Chaotic Elections!: A Mathematician Looks at Voting*, Providence, RI: American Mathematical Society, 2001.

Taylor, Alan D., *Mathematics and Politics: Strategy, Voting, Power and Proof*, New York: Springer-Verlag, 1995.

CHAPTER 7 FOUNDATIONS OF MEASUREMENT THEORY

The most comprehensive recent work on measurement theory is the three-volume study of David Krantz, R. Duncan Luce, Patrick Suppes, and Amos Tversky, *Foundations of Measurement*, New York: Academic Press, 1971, 1989, 1990. See also L. Pfanzagl, *Theory of Measurement*, New York: Wiley, 1968—especially for a treatment of bisection systems; Chapter 2; "Psychological measurement theory" of Clyde H. Coombs, Robyn M. Dawes, and Amos Tversky, *Mathematical Psychology: An Elementary Introduction*, Englewood Cliffs: Prentice-Hall, 1970; and Patrick Suppes and Joseph L. Zinnes, "Basic Measurement Theory," in R. Duncan Luce, Robert R. Bush, and Eugene Galanter, *Handbook of Mathematical Psychology*, Vol. I, New York: Wiley, 1963, 1–76.

For measurement theory aimed toward applications in sociology, consult P. Abell, "Measurement in Sociology," *Sociology* 2 (1968): 1–20; and 3 (1969): 397–411; Chapter 2 of James S. Coleman, *Introduction to Mathematical Sociology*, Glencoe: Free Press, 1964; and Chapter 7 of

Thomas J. Fararo, *Mathematical Sociology: An Introduction to Fundamentals*, New York: Wiley, 1973.

Significant earlier works are as follow:

Campbell, N. R., *Physics*: *The Elements*, London: Cambridge University Press, 1920, republished as *Foundations of Science*, New York: Dover, 1957; and *An Account of the Principles of Measurements and Calculations*, London: Longmans, Green, 1928.

Hölder, Otto, "Die axiome der quantitität und die lehre von mass," *Berichte Über Die Berhandlungen Der Könglich Sächsischen Gesellschaft Der Wissenschaften Zu Leipzig, Mathematisch-Physische Classe* 53 (1901): 1–64.

Luce, R. Duncan, "Semi-Orders and a Theory of Utility Discrimination," *Econometrica* 24 (1956): 178–191; and *Individual Choice Behavior*, New York: Wiley, 1959.

Scott, Dana, and Patrick Suppes, "Foundational Aspects of Theories of Measurement," *Journal of Symbolic Logic* 23 (1958): 113–128.

Suppes, Patrick, "A Set of Independent Axioms for Extensive Quantities," *Portugaliae Maihematica* 10 (1951): 163–172.

Stevens, S. S. (1946), "On the Theory of Scales of Measurement," *Science* 103: 677–680.

Stevens, S. S. (1951), "Mathematics, Measurement, and Psychophysics," in S. S. Stevens, ed., *Handbook of Experimental Psychology*, New York: Wiley, 1–49.

Stevens, S. S. (1959), "Measurement," in C. W. Churchman, ed., *Measurement: Definitions and Theories*, New York: Wiley, 18–36. Reprinted in G. M. Maranell, ed., *Scaling: A Sourcebook for Behavioral Scientists*, Chicago: Aldine, 1974, 22–41.

Stevens, S. S. (1968), "Measurement, Statistics, and the Schemapiric View," *Science*, 161: 849–856.

CHAPTER 8 INTRODUCTION TO UTILITY THEORY

Allais, Maurice, "Le comportement de l'homme rationnel devant Ie risque: critique des postulats et axiomes de I'ecole Americaine," *Econometrica* 21 (1953): 503–546.

Anand, Paul, *Foundations of Rational Choice under Risk*, Oxford: Oxford University Press, 2002.

Barberà, Salvador, Peter Hammond, and Christian Seidl, eds., *Handbook of Utility Theory*, Vols. I and II, New York: Springer, 2004, provide survey articles on recent developments. The focus of Volume I is basic theory, while Volume II concentrates on extensions and applications to other branches of economic theory. Chapters on experimental and empirical research on utility and on the history of utility theory will appear in Volume III.

Bell, D. E., H. Raiffa, and A. Tversky, Eds., *Decision Making: Descriptive, Normative, and Prescriptive Interactions*, New York: Cambridge University Press, 1988.

Bernoulli, Daniel, "Specimen theoriae novae de mensura sortis," *Commentarii Academiae Scientiarwn Imperialis Petropolitanae*, 1730, 1731, 1738; L. Sommer, trans., "Expositions of a New Theory of the Measurement of Risk," *Econometrica* 22 (1954).

Edwards, Ward, ed., *Utility Theories: Measurements and Applications*, New York: Springer, 1992.

Ellsberg, Daniel, "Classic and current notions of measurable utility," *Economic Journal* 64 (1954): 528–556.

Fishburn, Peter, *Utility Theory for Decision Making*, New York: Wiley, 1964.

Fishburn, Peter, *The Foundations of Expected Utility*, New York: Springer, 1982.

Gilboa, Itzhak, *Theory of Decision under Uncertainty*, Cambridge: Cambridge University Press, 2009.

Herstein, I. N., and J. Milnor, "An Axiomatic Approach to Measurable Utility," *Econometrica* 21 (1953): 291–297.

Isbell, J. R., "Absolute Games," in A. W. Tucker and R. D. Luce, eds., *Contributions to the Theory of Games*, Vol. IV, Princeton: Princeton University Press, 1959, 357–396.

Keeney, R. L., and H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, New York: Cambridge University Press, 1993.

Kreps, David M., *Notes on the Theory of Choice*, Boulder, CO: Westview Press, 1988.

Luce, R. Duncan, and Patrick Suppes, "Preference, Utility and Subjective Probability," in Luce, Bush, and Galanter, *Handbook of Mathematical Psychology*, Vol. III, New York: Wiley, 1963, Chapter 19.

Luce, R. Duncan, and L. Narens, "S. S. Stevens, Measurement, and Psychophysics," in P. R. Killeen and W. R. Uttal, eds., *Fechner Day 99: The End of Twentieth Century Psychophysics*, Tempe, AZ: The International Society for Psychophysics, 1999.

Mosteller, Frederick, and Philip Nogee, "An Experimental Measurement of Utility," *Journal of Political Economy* 59 (1951): 371–404.

Owen, Guillermo, *Game Theory*, Philadelphia: Saunders, 1969. The proofs of Theorems 4 and 6 are adapted from Chapter 6 of Owen's book.

Savage, Leonard J., *The Foundations of Statistics*, New York: Wiley, 1954, especially Chapter 5, "Utility."

Stevens, S. S., "Mathematics, Measurement and Psychophysics," *Handbook of Experimental Psychology* (New York: Wiley, 1951), 1–49; and "On the Theory of Scales of Measurement," *Science* 103 (1946): 677–680.

Von Neumann, John, and Oskar Morgenstern, *The Theory of Games and Economic Behavior*, commem. ed., Princeton, NJ: Princeton University Press, 2007.

Also see the references to Campbell and to Coombs, Dawes, and Tversky in Chapter 7.

CHAPTER 9 EQUILIBRIUM IN AN EXCHANGE ECONOMY

References on General Equilibrium Analysis

Arrow, Kenneth J., and Gerard Debreu, "Existence of an Equilibrium for a Competitive Economy," *Econometrica* 22 (1954): 265–290.

Arrow, Kenneth J., and F. H. Hahn, *General Competitive Analysis*, San Francisco: Holden-Day, 1972, especially pages 16–29.

Cornwall, Richard, *Introduction to the Use of General Equilibrium Analysis*, Amsterdam: North-Holland, 1984.

Debreu, Gerard, *Theory of Value*: *An Axiomatic Analysis of Economic Equilibrium*, New York: Wiley, 1959.

Kemeny, John, and J. L. Snell, "Market Stability: A Dynamic Model," Chapter IV in *Mathematical Models in the Social Sciences*, Boston: Ginn, 1962.

Nicholson, Walter, "The Existence of General Equilibrium Prices," in *Microeconomic Theory*: *Basic Principles and Extensions*, 2nd ed., Independence KY: South-Western, Cengage, 2004, 594–605.

Quirk, J., and R. Saposnik, *Introduction to General Equilibrium and Welfare Economics*, New York: McGraw-Hill, 1968.

Reinhard, John, "Abraham Wald's Equilibrium Existence Proof Reconsidered," *Economic Theory* 13 (1999): 417–428.

Scarf, H. E., *The Computation of Economic Equilibria*, New Haven: Yale University Press, 1973.

Wald, Abraham, "Über Einige Gleichungssysteme der Mathematischen Okonomie," *Zeitschrift für National Okonomie* 7 (1936): 637–670; trans. "On Some Systems of Equations of Mathematical Economics," *Econometrica* 19 (1951): 368–403.

Walras, Léon, *Eléments d'Economie Politique Pure*, Lausanne: Corbaz, 1874, 1877; *Elements of Pure Economics*, W. Jaffee, trans., Homewood, Illinois: Richard D. Irwin, Inc., 1954.

Ye Du, *Essays on the Computation of Economic Equilibria and Its Applications*, Ph.D. dissertation, University of Michigan Ann Arbor, http://deepblue.lib.umich.edu/bitstream/handle/2027.42/64821/duye_1.pdf?sequence=1.

Brouwer's Fixed Point Theorem

Brouwer, L. E. J., "Über Abbildung und Mannigfaltigkeiten," *Mathematische Annalen* 71 (1910): 97–115.

Courant, Richard and Herbert Robbins, *What Is Mathematics?* London: Oxford University Press, 1941, 251–255.

Henle, Michael, *A Combinatorial Introduction to Topology*, New York: Dover, 1994.

Milnor, John, "Analytic Proofs of the 'Hairy Ball Theorem' and the Brouwer Fixed Pont Theorem," *American Mathematical Monthly* 85 (1978): 521–524.

Rogers, C. A., "A Less Strange Version of Milnor's Proof of Brouwer's Fixed-Point Theorem," *American Mathematical Monthly* 87 (1980): 525–527.

Franklin, Joel, *Methods of Mathematical Economics*, New York: Springer-Verlag, 1980, Chapter III.

CHAPTER 10 ELEMENTARY PROBABILITY

Elementary Concepts of Probability

Olinick, Michael, "Chance", Chapter 8 in Walter Meyer, ed., *Principles and Practice of Mathematics*, New York: Key College, 1996.

Goldstein, Larry J., David I. Schneider, and Martha J. Siegel, *Finite Mathematics and Its Applications*, 9th ed., Prentice Hall, 2006. Almost any text with "Finite Mathematics" in the title will provide an elementary treatment of probability, but this is an especially well written one.

More Advanced and Complete Accounts

Feller, William, *An Introduction to Probability Theory and Its Applications*, New York: Wiley, 1957. A classic text, still in print more than half a century after its original publication.

Chung, Kai Lai, and Farid Aitsahlia, *Elementary Probability Theory*, New York: Springer, 2006.

Grinstead, Charles M., and J. Laurie Snell, *Introduction to Probability*, Providence, RI; American Mathematical Society, 1997.

Stirzaker, David, *Elementary Probability*, Cambridge: Cambridge University Press, 2003.

Special Topics

Feynman, Richard P., Robert B. Leighton, and Matthew Sands, *The Feynman Lectures on Physics*, Vol. 3 (Reading, Mass: Addison-Wesley, 1965).

Three accessible treatments of the probabilistic birth and death process can be found in the following:

Pollard, J. H., *Mathematical Models for the Growth of Human Populations*, Cambridge: Cambridge University Press, 1973, especially Chapter 5, "Simple Birth and Death Processes."

Haigh, John, *Probability Models*, New York: Springer, 2002.

Ross, Sheldon M., *Introduction to Probability Models*, 9th ed., Amsterdam: Elsevier-Academic Press, 2007, especially Chapter 6: "Continuous Time Markov Chains."

Artem S. Novozhilov, Georgy P. Karev, and Eugene V. Koonin, "Biological Applications of the Theory of Birth-and-Death Processes." *Briefings in Bioinformatics* 2006 7(1): 70–85 argue that these processes "are a simple, natural and formal framework for modeling a vast variety of biological processes such as population dynamics, speciation, genome evolution, including growth of paralogous gene families and horizontal gene transfer and somatic evolution of cancers."

The following six references are good entry points for a study of probability and the law:

Finkelstein, Michael O., *Quantitative Methods in Law*, New York: Free Press, 1978.

Kahneman, D., P. Slovic, and A. Tversky, *Judgment under Uncertainty: Heuristics and Biases*, Cambridge: Cambridge University Press, 1982.

Kaye, D. H., and Jonathan J. Koehler, "Can Jurors Understand Probabilistic Evidence?" *Journal of the Royal Statistical Society, Series A*, 154 (1991): 75–81.

Schneps, Leila, and Coralie Colmez, *Math on Trial: How Numbers Get Used and Abused in the Courtroom*, New York: Basic Books, 2013.

Thompson, W. C., and Schumann, E. L., "Interpretation of Statistical Evidence in Criminal Trials: The Prosecutor's Fallacy and the Defense Attorney's Fallacy," *Law and Human Behavior* 11 (1987): 167–187.

Tillers, P., and E. Green, eds., *Probability and Inference in the Law of Evidence*, Dordrecht: Kluwer, 1988.

Tribe, Lawrence, "Trial by Mathematics: Precision and Ritual in the Legal Process," *Harvard Law Review* 84 (1971): 1329–1393.

Another interesting place to search for relevant studies is *Jurimetrics: The Journal of Law*, *Science*, *and Technology*, published quarterly by the American Bar Association Section of Science and Technology Law and the Center for the Study of Law, Science, and Technology at the Sandra Day O'Connor College of Law, Arizona State University.

CHAPTER 11 MARKOV PROCESSES

General

Bharucha-Reid, A. T., *Elements of the Theory of Markov Processes and Their Applications*, New York: McGraw-Hill, 1960; Mineola, NY: Dover, 1997. A particularly good source for applications in the physical sciences.

Kemeny, John G., and J. Laurie Snell, *Finite Markov Chains*, New York: Van Nostrand, 1960. The basic theory and many applications to the social sciences. I have followed the notation and terminology of this book, so it will be easy for you to read. An updated edition has been published as *Finite Markov Chains: With a New Appendix "Generalization of a Fundamental Matrix,"* New York: Springer, 1983.

Lawler, Gregory, *Introduction to Stochastic Processes*, 2nd ed., Chapamn and Hall/CRC, 2006.

Ross, Sheldon, *Introduction to Probability Models*, 10th ed., New York: Academic Press, 2009.

David Stirzaker, *Stochastic Processes and Models*, Oxford University Press, 2005.

More Specific Applications

Bartholomew, David, *Stochastic Models for Social Processes*, New York: Wiley, 1967.

Bartlett, M. S., *Stochastic Population Models in Ecology and Epidemiology*, London: Methuen, 1960.

Blumen, I., M. Kogan, and P. J. McCarthy, *The Industrial Mobility of Labor as a Probability Process*, Ithaca: Cornell Studies of Industrial and Labor Relations, 1955, Vol. 6.

Bush, R. R., and C. F. Mosteller, *Stochastic Models for Learning*, New York: Wiley, 1955.

Cohen, Bernard P., *Conflict and Conformity: A Probability Model and Its Applications*, Cambridge: MIT Press, 1963.

Kao, Edward P. C., "Modeling the Movement of Coronary Patients within a Hospital by Semi-Markov Processes," *Operations Research* 22 (1974): 683–699.

Von Hilgers, Philipp, and Amy N. Langville, "The Five Greatest Applications of Markov Chains," in Langville, Amy N., and William J. Stewart, *MAM* 2006: *Markov Anniversary Meeting*, *An International Conference to Celebrate the 150th Anniversary of the Birth of A. A. Markov*, Raleigh, North Caroline: Boson, 2006.

CHAPTER 12 TWO MODELS OF CULTURAL STABILITY

Beckingham, C. F., and G. W. B. Huntingford, trans. and ed., *Some Records of Ethiopia 1593–1646 Being Extracts from* The History of High Ethiopia or Assyria *by Manuel de Almeida Together with Bahrey's* History of the Galla, London: Hakluyt Society, 1954.

Gumii Bilisummaa Oromiyaa (Oromiyaa Liberation Council), "Gadaa System," at www.gumii.org.

Hoffmann, Hans, "Formal versus Informal Estimates of Cultural Stability," *American Anthropologist* 67 (1965): 110–115. Used with permission.

Hoffmann, Hans, "Markov Chains in Ethiopia," in Paul Kay, ed., *Explorations in Mathematical Anthropology*, Cambridge: MIT Press, 1971, 181–190. Used with permission.

Hoffmann, Hans, "Mathematical Anthropology," in Bernard J. Siegel, ed., *Biennial Review of Anthropology*, Stanford, CA: Stanford University Press, 1969.

Inside BU, "Hoffmann Brought His 'Fascination of Anthropology' to the Classroom," March 13, 1997, www.binghamton.edu/inside/March-April/MARCH-13-97/Hoffmann.html.

Legesse, Asmarom, *Gada: Three Approaches to the Study of African Society*, New York: Free Press, 1973.

Murdock, George Peter, *Africa: Its Peoples and Their Cultural History*, New York: McGraw-Hill, 1959, 323–327.

Prins, Adriaan Hendrik Johan, *East African Age-Class Systems*, Groningen, Djakarta: Wolters, 1953.

Strouthes, Daniel "Hans Hoffmann," *Miscellaneous Obituaries of Anthropologists*, www.obitcentral.com/obitsearch/obits/misc/anthro2.htm.

For a glimpse at some research in mathematical modeling in anthropology that came after Hoffmann's studies, see the following:

De Meur, Gisele, *New Trends in Mathematical Anthropology*, Boston: Routledge and Kegan Paul, 1986.

Chibnik, Michael, "The Use of Statistics in Sociocultural Anthropology," *Annual Review of Anthropology* 14 (1985): 135–157.

The online journal *Mathematical Anthropology and Cultural Theory*, www.mathematicalanthropology.org.

CHAPTER 13 PAIRED-ASSOCIATE LEARNING

Atkinson, Richard C., Gordon H. Bower, and Edward J. Crothers, *An Introduction to Mathematical Learning Theory*, New York: Wiley, 1965.

Bower, Gordon H., "Application of a Model to Paired-Associate Learning," *Psychometrika* 26 (1961): 255–280. Used with permission.

Bower, Gordon H., and Ernest R. Hilgard, *Theories of Learning*, Englewood Cliffs: Prentice Hall, 1981.

Bush, Robert R., and Frederick A. Mosteller, "A Comparison of Eight Models," in R. R. Bush and W. K. Estes, eds., *Studies in Mathematical Learning Theory*, Stanford: Stanford University Press, 1959, 293–307.

Bush, Robert R., and Saul Sternberg, "A Single-Operator Model," in R. R. Bush and W. K. Estes, eds., *Studies in Mathematical Learning Theory*, Stanford: Stanford University Press, 1959, 204–214.

Calkins, Mary, "Association," in Hugo Münsterberg et al., "Studies from the Harvard Psychological Laboratory II," *Psychological Review* 1 (1894): 476–483.

Calkins, Mary, "Autobiography of Mary Whiton Calkins," in Carl Murchison, *A History of Psychology in Autobiography*, Worcester: Clark University Press, 1930.

Furumoto, Laurel, "Mary Whiton Calkins (1863–1930)," *Psychology of Women Quarterly* 5 (1980): 55–68.

Gluck, M. A., J. R. Anderson, and S. M. Kosslyn, eds., *Memory and Mind: A Festschrift for Gordon H. Bower*, New Jersey: Lawrence Erlbaum Associates, 2007.

Holyoak, Keith J., and Robert G. Morrison, eds., *The Cambridge Handbook of Thinking and Reasoning*, New York: Cambridge University Press, 2005.

Krantz, David, ed., *Contemporary Developments in Mathematical Psychology*, San Francisco: Freeman, 1974.

Restle, Frank, and James G. Greeno, *Introduction to Mathematical Psychology*, Reading, MA: Addison-Wesley, 1970.

Scarborough, Elizabeth, and Laurel Furumoto, *Untold Lives: The First Generation of American Women Psychologists*, New York: Columbia University Press, 1987.

CHAPTER 14 EPIDEMICS

Epidemics and History

Cartwright, Frederick F., *Disease and History*, New York: Crowell, 1972.

Defoe, Daniel, *A Journal of the Plague Year*, London: Oxford University Press, 1969.

Gallup, Joseph A., *Sketches of Epidemic Disease in the State of Vermont*, Boston: Wait and Sons, 1815.

Hays, J. N., *The Burdens of Disease: Epidemics and Human Response in Western History*, New Brunswick, NJ: Rutgers University Press, 1998.

Mack, Arien, ed., *In Time of Plague: The History and Social Consequences of Lethal Epidemic Disease*, New York: New York University Press, 1991.

McNeill, William H., *Plagues and Peoples*, Garden City, NY: Doubleday, 1976.

Thucydides, *History of the Peloponnesian War,* trans. B. Jowett, in F. R. B. Godolphin, ed., Greek Historians, Vol. I, New York: Random House, 1942.

Watts, Sheldon, *Epidemics and History: Disease, Power, and Imperialism*, New Haven, CT: Yale University Press, 1997.

Zinsser, Hans, *Rats, Lice and History*, 4th ed., London: Routledge, 1942.

Texts and Papers

Bailey, Norman T. J., "A Simple Stochastic Epidemic," *Biometrika* 37 (1950): 193–202.

Hamer, W. H., "Epidemic Disease in England," *Lancet* 1 (1906): 723–739.

Kermack, W. D., and A. G. McKendrick, "A Contribution to the Mathematical Theory of Epidemics," *Journal of the Royal Statistical Society* 115 (1927): 700–721.

McKendrick, A. G., "Applications of Mathematics to Medical Problems," *Proceedings of the Edinburgh Mathematical Society* 44 (1926): 98–130.

Mickens, Ronald E., "A SIR-Model with Square-Root Dynamics: An NSFD Scheme," *Journal of Difference Equations and Applications* 16 (2010): 209–216.

Mickens, Ronald E., "An Exactly Solvable Model for the Spread of Disease," *The College Mathematics Journal* 43 (2012): 114–121.

Munz, Philip, Ioan Hudea, Joe Imad, and Robert J. Smith, "When Zombies Attack!: Mathematical Modelling of an Outbreak of Zombie Infection," *Infectious Disease Modelling Research Progress* (2007): 133–150.

Noymer, Andrew, "The Transmission and Persistence of 'Urban Legends': Sociological Application of Age-Structured Models," *Journal of Mathematical Sociology* 25 (2001): 299–323.

Ross, R., *The Prevention of Malaria*, 2nd ed., London: Murray, 1911.

Saunders, I. W., "A Model for Myxomatosis," *Mathematical Biosciences* 45 (1980): 1–15.

Soper, H. E., "Interpretation of Periodicity in Disease-Prevalence," *Journal of Royal Statistical Society* 92 (1929): 34–73.

Texts and Survey Articles

Abdul-Razak, Abdul-Karim, "Modeling Domestic Violence and Predicting Its Growth Using Differential Equations, a Case Study of Women and Children in Tamale," M.Sc. thesis in industrial mathematics, Kwame Nkrumah University of Science and Technology, 2012.

Anderson, Roy M., and Robert M. May, *Infectious Diseases of Humans: Dynamics and Control*, Oxford: Oxford University Press, 1999.

Bailey, Norman T. J., "Approaches to the Control of Infectious Disease," in J. Gani, ed., *Perspectives in Probability and Statistics*, London: Applied Probability Trust, 1975.

———, *The Mathematical Theory of Epidemics*, London: Griffin, 1957.

———, *The Mathematical Theory of Infectious Diseases*, London: Griffin, 1975.

Bartlett, Maurice S., *Stochastic Population Models in Ecology and Epidemiology*, London: Methuen, 1960.

Castillo-Chavez, Carlos, and Baojun Song, "Models for the Transmission Dynamics of Fanatic Behaviors," 155–172 in Banks and Castillo-Chavez, eds., *Bioterrorism: Mathematical Modeling Applications in Homeland Security*, Philadelphia: SIAM, 2003.

Cintróán-Arias, Ariel, Fabio Sánchez, Xiaohong Wang, Carlos Castillo-Chavez, Dennis M. Gorman, and Paul J. Gruenewald, "The Role of Nonlinear Relapse on Contagion Amongst Drinking Communities," in Gerardo Chowel et al., eds., *Mathematical and Statistical Estimation Approaches in Epidemiology*, New York: Springer, 2009.

Daley, Daryl J., Joseph M. Gani, *Epidemic Modeling: An Introduction*, Cambridge, UK: Cambridge University Press, 1999.

De Hoog, F., Joseph Gani, and David Gates, "A Threshold Theorem for the General Epidemic Model," *Journal of Mathematical Biology* 8 (1979): 113–121.

Dietz, K., "Epidemics and Rumours: A Survey," *Journal of Royal Statistical Society* 130 (1967): 505–528.

González, Beverly, Emilia Huerta-Sánchez, Angela Ortiz-Nieves, Terannie Vázquez-Alvarez, and Christopher Kribs-Zaleta, C., "Am I Too Fat? Bulimia as an Epidemic," *Journal of Mathematical Psychology* 47 (2003): 515–526.

Poole, Robert W., "Epidemics and Disease," in *Introduction to Quantitative Ecology*, New York: McGraw-Hill, 1974.

Sánchez, Fabio, Xiaohong Wang, Carlos Castillo-Chávez, Dennis M. Gorman, and Paul J. Gruenewald, "Drinking as an Epidemic: A Simple Mathematical Model with Recovery and Relapse," in K. Witkiewitz and G. Marlatt, eds., *Therapist's Guide to Evidence-Based Relapse Prevention*, New York: Academic Press, 2007.

Smith, Robert, *Modelling Disease Ecology with Mathematics*, Palo Alto, CA: American Institute of Mathematical Sciences, 2008.

Spicer, Clive, "The Mathematical Modeling of Influenza Epidemics," *British Medical Bulletin* 35 (1979): 23–28.

Waltman, Paul, *Deterministic Threshold Models in the Theory of Epidemics*, New York: Springer-Verlag, 1974.

CHAPTER 15 ROULETTE WHEELS AND HOSPITAL BEDS: A COMPUTER SIMULATION OF OPERATING AND RECOVERY ROOM USAGE

Banks, Jerry, ed., *Handbook of Simulation: Principles, Methodology, Advances, Applications and Practice*, New York: Wiley, 1998.

Banks, Jerry, John Carson, Barry Nelson, and David Nicol, *Discrete-Event System Simulation*, 5th ed., New York: Prentice Hall, 2005.

Bratley Paul, Bennett L. Fox, and Linus E. Schrage, *A Guide to Simulation*, 2nd ed., New York: Springer, 1987.

Chung, Christopher A., *Simulation Modeling Handbook: A Practical Approach*, Boca Raton, FL: CRC Press, 2003.

Delaney, William, and Erminia Vacari, *Dynamic Models and Discrete Event Simulation*, New York: Marcel Dekker, 1988.

Dyson, George, *Turing's Cathedral: The Origins of the Digital Universe*, New York: Pantheon, 2012.

Fetter, R. B., and J. D. Thompson, "The Simulation of Hospital Systems," *Operations Research* 13 (1965): 689–711.

Fishwick, Paul, *Handbook of Dynamic System Modeling*, Boca Raton, FL: Chapman and Hall/CRC, 2007.

Gaylord, Richard J., and Louis J. D'Andria, *Simulating Society: A Mathematica Toolkit for Modeling Socioeconomic Behavior*, New York: Springer-Verlag, 1998.

Gilbert, Nigel, and Klaus G. Troitzsch, *Simulation for the Social Scientist*, 2nd ed., Philadelphia: Open University, 2011.

Gilbert, Nigel, ed., *Computer Simulation in the Social Sciences*, Thousand Oaks, CA: SAGE, 1999.

Gilbert, Nigel, and Rosario Conte, *Artificial Societies: The Computer Simulation of Social Life*, London: University College London Press, 1995.

Grimmett, Geoffrey, and David Stirzaker, *Probability and Random Processes*, Oxford: Oxford University Press, 2002.

Guetzkow, Harold, Philip Kotler, and Randall L. Schultz, eds., *Simulation in Social and Administrative Science*, Englewood Cliffs, NJ: Prentice-Hall, 1972.

Law, Averill, *Simulation Modeling and Analysis*, 4th ed., New York: McGraw-Hill, 2006.

McHaney, Roger W., *Computer Simulation: A Practical Perspective*, San Diego: Academic Press, 1991.

Metropolis, Nicholas, and Stanislaw Ulam, "The Monte Carlo Method," *Journal of the American Statistical Association* 44 (1949): 335–341.

Nutaro, James J., *Building Software for Simulation: Theory and Algorithms, with Applications in C++*, New York: Wiley, 2010.

Pidd, Michael, *Computer Simulation in Management Science*, 4th ed., New York: Wiley, 1998.

Schmitz, Homer H., and N. K. Kwak, "Monte Carlo Simulation of Operating-Room and Recovery-Room Usage," *Operations Research* 20 (1972): 1171–1180. Used with permission of the Operations Research Society of America and the authors.

Schmitz, Homer H., N. K. Kwak, and P. J. Kuzdrall, "The Determination of Surgical Suite Capacity and an Evaluation of Patient Scheduling Policies," Recherche Opérationnelle 12 (1978): 3–14.

Smith, John, *Computer Simulation Models*, New York: Hafner, 1968.

Ulam, Stanislaw, *Adventures of a Mathematician*, New York: Scribner's, 1976.

Winsburg, Eric, *Science in the Age of Computer Simulation*, Chicago: University of Chicago Press, 2010.

Zeigler, Bernard P., Herbert Praehofer, and Tag Gon Kim, *Theory of Modeling and Simulation: Integrating Discrete Event and Continuous Complex Dynamic Systems*, 2nd ed., San Diego: Academic Press, 2000.

Pseudorandom Number Generators

Barker, Elaine, and John Kelsey, *Recommendation for Random Number Generation Using Deterministic Random Bit Generators*, National Institute of Standards and Technology Publication SP800-90A, January 2012.

Luby, Michael *Pseudorandomness and Cryptographic Applications*, Princeton, NJ: Princeton University Press, 1996.

Knuth, Donald, *The Art of Computer Programming*, vol. 2: *Seminumerical Algorithms*, 3rd ed., Reading, MA: Addison-Wesley, 1997.

For a recent paper using analytic and simulation techniques to investigate a hospital problem using a systems of nonlinear differential equations, see Lamooki, Gholam, Reza Rokni Lamooki, Farzaneh Maleki, and Amirhossein Hajihosseini, "A Mathematical Model for the Admission Process in Intensive Care Units," *Communications in Nonlinear Science and Numerical Simulation* (2013).

CHAPTER 16 GAME THEORY

Aumann, Robert, 1974, "Subjectivity and Correlation in Randomized Strategies," *Journal of Mathematical Economics* 1 (1974): 67–96.

Aumann, Robert, "Correlated Equilibrium as an Expression of Bayesian Rationality," *Econometrica* 55 (1987): 1–18.

Aumann, Robert, and Michael Maschler, "Game Theoretic Analysis of a Bankruptcy Problem in the Talmud," *Journal of Economic Theory* 36 (1985): 195–213.

Aumann, Robert, and Michael Maschler, *Repeated Games with Incomplete Information*, Cambridge: MIT Press, 1995.

Axelrod, Robert, *The Evolution of Cooperation*, New York: Basic Books, 2006.

Behn, Robert D., and James W. Vaupel, *Quick Analysis for Busy Decision Makers*, New York: Basic Books, 1982.

Brams, Steven, *Biblical Games: Game Theory and the Hebrew Bible*, Cambridge: MIT Press, 1980.

Brams, Steven, *Superior Beings: If They Exist, How Would We Know? Game-Theoretic Implications of Omniscience,* *Omnipotence, Immortality, and Incomprehensibility*, New York: Springer, 2007.

Dresher, Melvin, *Games of Strategy: Theory and Applications*. Englewood Cliffs, NJ: Prentice-Hall, 1961.

Dyson, George, *Turing's Cathedral: The Origins of the Digital Universe*, Vintage 2012.

Jones, Antonia J., *Game Theory: Mathematical Models of Conflict,* Chichester: Horwood Publishing, 2000.

Krugman, Paul, "Is Free Trade Passé?" *Journal of Economic Perspectives* 1 (1987): 131–144.

Maynard Smith, John, *Evolution and the Theory of Games*, Cambridge, UK: Cambridge University Press, 1982.

Morgenstern, Oskar, "The Collaboration between Oskar Morgenstern and John von Neumann on the Theory of Games," *Journal of Economic Literature* 14 (1976): 805–816.

Nash, John, Harold William Kuhn, and Sylvia Nasar, *The Essential John Nash*, Princeton: Princeton University Press, 2007 (contains Nash's papers on equilibrium and a copy of his Ph.D. thesis).

Nash, John, "The Bargaining Problem," *Econometrica* 18 (1950): 155–162.

Nasar, Sylvia, *A Beautiful Mind: A Biography of John Forbes Nash, Jr., Winner of the Nobel Prize in Economics*, New York: Simon & Schuster, 1998.

O'Neill, Barry, "A Problem of Rights Arbitration from the Talmud," *Mathematical Social Sciences* 2 (1982): 345–371.

Rapoport, Anatol, *Fights, Games and Debates*, Ann Arbor: University of Michigan Press, 1960.

Rapoport, Anatol, N-*Person Game Theory: Concepts and Applications*, Ann Arbor: University of Michigan Press, 1970.

Rapoport, Anatol, *Two-Person Game Theory: The Essential Ideas*, Ann Arbor: University of Michigan Press, 1966.

Rapoport, Anatol, and Albert Chammah, *Prisoner's Dilemma: A Study in Conflict and Cooperation*, Ann Arbor: University of Michigan Press, 1965.

Shapley, Lloyd, "A Value for *n*-Person Games," in H.W. Kuhn and A.W. Tucker, eds., *Contributions to the Theory of Games*, vol. II, *Annals of Mathematical Studies* v. 28, 307–317. Princeton, NJ: Princeton University Press, 1953.

Shapley, Lloyd, and Martin Shubik, "A Method for Evaluating the Distribution of Power in a Committee System," *American Political Science Review* 48 (1954): 787–792.

Siegfried, Tom, *A Beautiful Math: John Nash, Game Theory, and the Modern Quest for a Code of Nature*, Washington, D.C.: Joseph Henry Press, 2006.

Sorin, Sylvain, *A First Course on Zero-Sum Repeated Games*, Springer, 2002.

Straffin, Philip D., *Game Theory and Strategy*, Washington, D.C.: Mathematical Association of America, 1993.

Von Neumann, John, "Zur Theorie der Gesellschaftsspiele," *Mathematische Annalen* 100 (1928): 295–320. (Trans. "On the Theory of Games of Strategy," in by A.W. Tucker and R. D. Luce, eds., *Contributions to the Theory of Games*, vol. IV, Annals of Mathematics Studies 40: 13–42), Princeton, NJ: Princeton University, 1959.

Von Neumann, John and Oskar Morgenstern, *Theory of Games and Economic Behavior*, Princeton, NJ: Princeton University Press, 1944.

CHAPTER 17 RECIDIVISM IN THE CRIMINAL JUSTICE SYSTEM

Banks, Harvey T., and Carlos Castillo-Chavez, *Bioterrorism: Mathematical Modeling of Applications in Homeland Security*, Philadelphia: SIAM, 1987.

Blumstein, Alfred, and Richard Larson, "Problems in Modeling and Measuring Recidivism," *Journal of Research in Crime and Delinquency* 8 (1971): 124–132.

Brent, Edward E., and Richard E. Sykes, "A Mathematical Model of the Process of Symbolic Interaction between Police and Suspects," *Behavioral Science* 24 (1979): 388–402.

Brent, Edward E., and Richard E. Sykes, "The Regulation of Interaction by Police: A Systems View of Taking Charge." *Criminology* 18 (1980): 182–197

Greenberg, David F., *Mathematical Criminology*, New Bruswick, NJ: Rutgers University Press, 1979.

Maltz, Michael, *Recidivism*, New York: Academic Press, 1984.

Latessa, Edward J., Shelley J. Listwan, and Deborah Koetzle, *What Works (and Doesn't) in Reducing Recidivism*, Waltham, MA: Anderson: 2013.

MacKenzie, Doris Layton, *What Works in Corrections: Reducing the Criminal Activities of Offenders and Delinquents*, Cambridge, UK: Cambridge University Press, 2006.

Memon, N., J. D. Farlet, D. L. Hicks, and T. Rosenorm, *Mathematical Methods in Counterterrorism*, New York: Springer, 2009.

Pettiway, Leon E., Sergey Dolinsky, and Alexander Grigoryan, "The Drug and Criminal Activity Patterns of Urban Offenders: A Markov Chain Analysis," *Journal of Quantitative Criminology*, 10 (1994): 79–107.

Rardin, Ronald L., and Paul Gray, "Analysis of Crime Control Strategies," *Journal of Criminal Justice* 1 (1973): 339–346.

Rey, Sergio, Elizabeth A. Mack, and Julia Koschinsky, "Exploratory Space–Time Analysis of Burglary Patterns," *Journal of Quantitative Criminology* 28 (2012): 509–531.

Schmidt, Peter, and Ann Dryden Witte, *Predicting Recidivism Using Survival Models*, New York: Springer-Verlag, 1988.

Stander, Julian, David P. Farrington, Gillian Hill, and Patricia M. E. Altham, "Markov Chain Analysis and Specialization in Criminal Careers," *British Journal of Criminology* 29 (1989): 317–335.

Syed, Feroz Shah, Didong Li, Xun Zhang, and Zhenhong Guo, "Mathematical Modelling in Criminology," *Malaysian Journal of Mathematical Sciences* 7 (2013): 125–145.

White, David, Soo Hong Uh, and Kim Andriano, "Juvenile Court Records and Markov Chains: Their Use as Aids in Identification and Treatment Of Delinquent Youth," *Law and Human Behavior* 1 (1977): 217–237.

Zamble, Edward, and Vernon L. Quinsey, *The Criminal Recidivism Process*, Cambridge, UK: Cambridge University Press, 2001.

APPENDIX V DIFFERENTIAL EQUATIONS

Boyce, William E., and Richard C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, 10th ed., New York: Wiley, 2012.

Hubbard, John H., and Beverly H. West, *Differential Equations: A Dynamical Systems Approach*, *Part I: Ordinary Differential Equations*, New York: Springer-Verlag, 1997.